Homework Take-Up:

5c) $y = (x-5)^{1/3}$

$y' = \frac{1}{3}(x-5)^{-2/3}$

$= \frac{1}{3(x-5)^{2/3}}$ ←

$= \frac{1}{3\sqrt[3]{(x-5)^2}}$

$y' = \frac{1}{3\sqrt[3]{(x-5)^2}}$

$0 = \frac{1}{3\sqrt[3]{(x-5)^2}}$

$y'' = -\frac{2}{9}(x-5)^{-\frac{5}{3}}$

$= \frac{-2}{9\sqrt[3]{(x-5)^5}}$

* check later!

7e) $f(x) = \sqrt{x^2 - 2x + 2}$

$$f'(x) = \frac{1}{2}(x^2 - 2x + 2)^{-1/2}(2x - 2)$$

$$= \frac{x - 1}{\sqrt{(x^2 - 2x + 2)}}$$

	$x < 1$	$x > 1$
$x - 1$	$-$	$+$
$(x^2 - 2x + 2)^{\frac{1}{2}}$	$+$	$+$
$f'(x)$	$-$	$+$

$\therefore$ minimum at $f(1) = 1$.

L3. (4.3) Limits and Asymptotes

- Horizontal Asymptotes: Limits as $x \to \pm\infty$

Ex1:Take & Compare the following limits, if they exist.

FROM MHF H.A. $y=0$

$$\lim_{x\to\infty}\frac{1}{x^2+1} \qquad \lim_{x\to-\infty}\frac{1}{x^2+1}$$

$= 0$ $\qquad$ $= 0$

We can find horizontal asymptotes easily now.

Ex2:Determine the following limits, if they exist.

OBLIQUE

$$\lim_{x\to\infty}\frac{x^2}{x+2}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} \qquad \lim_{x\to-\infty}\frac{x^2}{x+2}$$

$= \lim\limits_{x\to\infty}\dfrac{1}{\frac{1}{x}+\frac{2}{x^2}}$ $\qquad$ $=$ DNE

$=$ undefined DNE

Ex3:Determine the following limits, if they exist. Explain the outcome.

$$\lim_{x\to\infty}\frac{-x^2}{x^2+5}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} \qquad \lim_{x\to-\infty}\frac{-x^2}{x^2+5}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$= \lim\limits_{x\to\infty}\dfrac{-1}{1+\frac{5}{x^2}}$ $\qquad$ $= \lim\limits_{x\to-\infty}\dfrac{-1}{1+\frac{5}{x^2}}$

$= -1$ $\qquad$ $= -1$

$\therefore y=-1$ is the horizontal asymptotes.

Horizontal Asymptotes and Limits at Infinity

If $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$, we say that the line $y = L$ is a horizontal asymptote of the graph $f(x)$.

Ex4:Evaluate

a) $\lim_{x\to\infty} \frac{x}{x^2+2x+1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{\frac{1}{x}}{1+\frac{2}{x}+\frac{1}{x^2}}$

$= \frac{0}{1+0+0}$

$= 0$

b) $\lim_{x\to\infty} \frac{2x^3+x^2}{1-3x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}}$

$= \lim_{x\to\infty} \frac{2+\frac{1}{x}}{\frac{1}{x^3}-3}$

$= -\frac{2}{3}$

Vertical Asymptotes of Rational Functions

A rational function of the form $f(x) = \frac{p(x)}{q(x)}$ has a vertical asymptote $x = c$ if $q(c) = 0$ and $p(c) \neq 0$.

Ex5:Evaluate

$$\lim_{x \to 4} \frac{3}{4-x}$$

Left sided:

$$\lim_{x \to 4^-} \frac{3}{4-x}$$

choose $x = 3.9999$
and solve

$= +\infty$

$\therefore$ DNE

Right sided limits:

$$\lim_{x \to 4^+} \frac{3}{4-x}$$

choose $x = 4.00001$
and take limit

$= -\infty$

$\therefore$ DNE

Assigned Work

p.193 #3, 4, 5, 7, 9, 10ace, 12