Assessment Quiz 2 Unit 7

1. Find the Cartesian equation for a line passing through points (8,5) and (-6,2).

2. Determine the acute angle between the lines to the nearest degree.

$x - 3y + 6 = 0$ & $-2x + y - 8 = 0$

L3(8.3) The Vector, Parametric, & Symmetric Equations of a Line in $\mathbf{R}^3$

The vector equation of a line in 3D is still:

$$\vec{r} = \vec{r}_0 + t\vec{m}$$

but now it looks like: $\vec{r} = (x_0, y_0, z_0) + t(a,b,c)$

Based on this formula, what are the parametric equations for a line in 3D?

$$x = x_0 + at \rightarrow t = \frac{x - x_0}{a}$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

Symmetric Equation of a Line in 3D

Solving each of the parametric equations for t gives:

$$t = \frac{x - x_0}{a} \qquad t = \frac{y - y_0}{b} \qquad t = \frac{z - z_0}{c}$$

These expressions give an alternate form for equations of straight lines in space:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

→ ONLY IN 3D

Where (x_0, y_0, z_0) are the coordinates of some point on the line

and $\vec{m} = (a, b, c)$ is the direction vector of the line

Ex1: a) What is the direction vector of each line?
b) State the coordinates of a point on the line.

i) $\frac{x-2}{7} = y+5 = \frac{z}{3}$

$\vec{m} = (7, 1, 3)$

point = $(2, -5, 0)$

ii) $\frac{x+4}{2} = \frac{-y+1}{1} = \frac{6-z}{7}$

$\vec{m} = (2, -1, -7)$

pt. = $(-4, 1, 6)$

Ex2: a) Write a vector equation of the line defined by
-x = y + 2 = z.
b) State three coordinates on the line. Which equation did you use? Why?

(a) $\vec{r} = (0, -2, 0) + t(-1, 1, 1)$, $t \in \mathbb{R}$

(b) $(-1, -1, 1)$, $t = 1$

$(0, -2, 0)$, $t = 0$

$(-2, 0, 2)$, $t = 2$

Use Vector because easier.

Ex3: Do the following equations represent the same line?

① $\frac{x-5}{2} = \frac{y+4}{-5} = \frac{z+1}{3}$ | ② $\frac{x+1}{-4} = \frac{y-11}{10} = \frac{z+4}{-6}$

Step 1: Verify that the direction vectors are collinear.
Step 2: Check to see if the point from line 1 is on line 2.

Step 1: $\vec{m}_1 = (2,-5,3)$ $\vec{m}_2 = (-4,10,-6)$

$\therefore -2\vec{m}_1 = \vec{m}_2$

$\therefore$ vectors are collinear.

What do you notice about the values of a and b?

Step 2: $(x_0, y_0, z_0) = (5,-4,-1)$ check:

$\frac{x+1}{-4}$ | $\frac{y-11}{10}$ | $\frac{z+4}{-6}$

$= \frac{5+1}{-4}$ | $= \frac{-15}{10}$ | $= \frac{-1+4}{6}$

$= -\frac{3}{2}$ | $= -\frac{3}{2}$ | $= -\frac{1}{2}$

$\therefore$ Since the point is not consistent the lines are parallel & not coincident.

Ex4: Find the parametric and symmetric equations of the line passing through the points A(-5, 2, 1) and B(3, 2, -4).

$\vec{m} = (8, 0, -5)$

PARAMETRIC

$x = -5 + 8t$

$y = 2$

$z = 1 - 5t$

Symmetric

$\frac{x+5}{8} = \frac{z-1}{-5}$, $y = 2$

Assigned Work:

p.449-450 #1bcd, 2bcd,
4abc (part c good question!),
5bde, 6, 9,10d, 12