Lesson #3: Rational Functions of the Form: $f(x) = \dfrac{\mathrm{a}x+\mathrm{b}}{\mathrm{c}x+\mathrm{d}}$

Example #1: Determine the key features of the graph to help sketch the function: $f(x) = \dfrac{3x+2}{4x-1}$

Critical Properties of Rational Functions:

x-intercepts:
For x intercept, let f(x) = 0, and solve for x:

$0 = \dfrac{3x+2}{4x-1}$ so $0 = 3x+2$

$-\dfrac{2}{3} = x$

y-intercepts:
For y intercept, let x = 0, and solve for y: (or $f(x)$)

$f(x) = \dfrac{3(0)+2}{4(0)-1}$

$= -2$

Vertical Asymptotes:
The equation of the vertical asymptote can be found by setting the denominator equal to zero and solving for x (providing the numerator does not have the same zero).

$4x-1=0$

$x = \dfrac{1}{4}$

Vertical asymptote is $x = \dfrac{1}{4}$

Horizontal Asymptotes:
The equation of the horizontal asymptote can be found by dividing each term in both the numerator and the denominator by x, and by investigating the end behavior of the function

$f(x) = \dfrac{\frac{3x}{x} + \frac{2}{x}}{\frac{4x}{x} - \frac{1}{x}}$

$= \dfrac{3 + \frac{2}{x}}{4 - \frac{1}{x}}$

as $x \to \pm\infty$

$\frac{2}{x} \to 0$

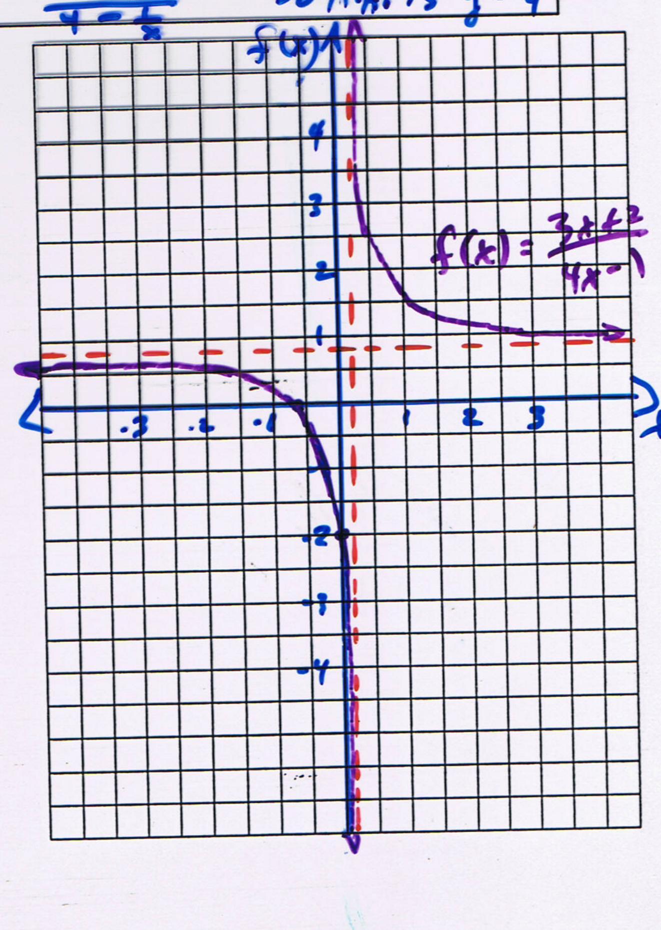
$\frac{1}{x} \to 0$

so H.A. is $y = \dfrac{3}{4}$

Domain: $D = \{x \mid x \in \mathbb{R}, x \neq \frac{1}{4}\}$

Range: $R = \{y \mid y \neq \frac{3}{4}, y \in \mathbb{R}\}$

Sketch the function:



Note:
If the degree of the numerator is equal to the degree of the denominator,
then the horizontal asymptote is: $\dfrac{a}{c}$

Since the degrees are equal then, the horizontal asymptote is: $y = \dfrac{3}{4}$