KEY CONCEPTS

- To solve **rational equations** algebraically, start by factoring the expressions in the numerator and denominator to find asymptotes and restrictions.

- Next, multiply both sides by the factored denominators, and simplify to obtain a polynomial equation. Then solve.

- For **rational inequalities,**
  - It can often help to rewrite with the right side equal to 0. Then, use test points to determine the sign of the expression in each interval.
  - If there is a restriction on the variable, you may have to consider more than one case.

  For example, if $\frac{a}{x-k} < b$, case 1 is $\mathbf{x > k}$ and case 2 is $\mathbf{x < k}$.

Example #3: Solve: $\frac{x^2-3x-4}{x^2+11x+30} \geq 0$

Factor the rational expression:

$\frac{(x-4)(x+1)}{(x+5)(x+6)} \geq 0$

Find the x-intercepts:

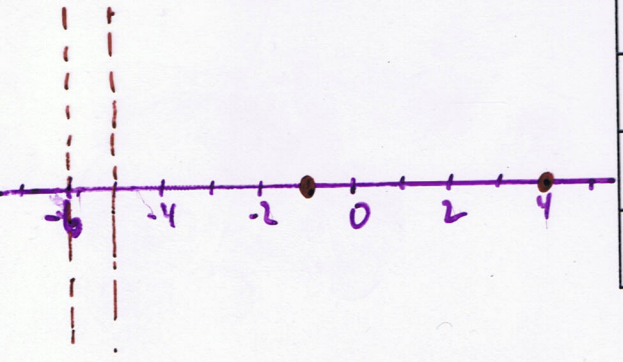
x intercepts are $4, -1$

State the restrictions:

$x \neq -5$ or -6

The function may change sign at the x-intercepts or at the vertical asymptotes.
Thus, these values will be used to determine the intervals of the function

Interval	Test Value for x	Sign of Factors of:	Sign of Function
$x < -6$	$x = -7$	$\frac{(-)(-)}{(-)(-)}$	+
$-6 < x < -5$	$x = -5.5$	$\frac{(-)(-)}{(-)(+)}$	−
$-5 < x \leq -1$	$x = -2$	$\frac{(-)(-)}{(+)(+)}$	+
$x = -1$	$x = -1$	$\frac{(-)(0)}{(+)(+)}$	NO SIGN
$-1 < x < 4$	$x = 2$	$\frac{(-)(+)}{(+)(+)}$	−
$x = 4$	$x = 4$	$\frac{(0)(+)}{(+)(+)}$	NO SIGN
$x > 4$	$x = 6$	$\frac{(+)(+)}{(+)(+)}$	+



∴ solutions are:

$x < -6, \; -5 < x \leq -1, \; x \geq 4$

Homework: pg 183; # 2, (3 – 5)odd, 6 – 8, (9, 10)odd, 12
Handin: pg 185; #17