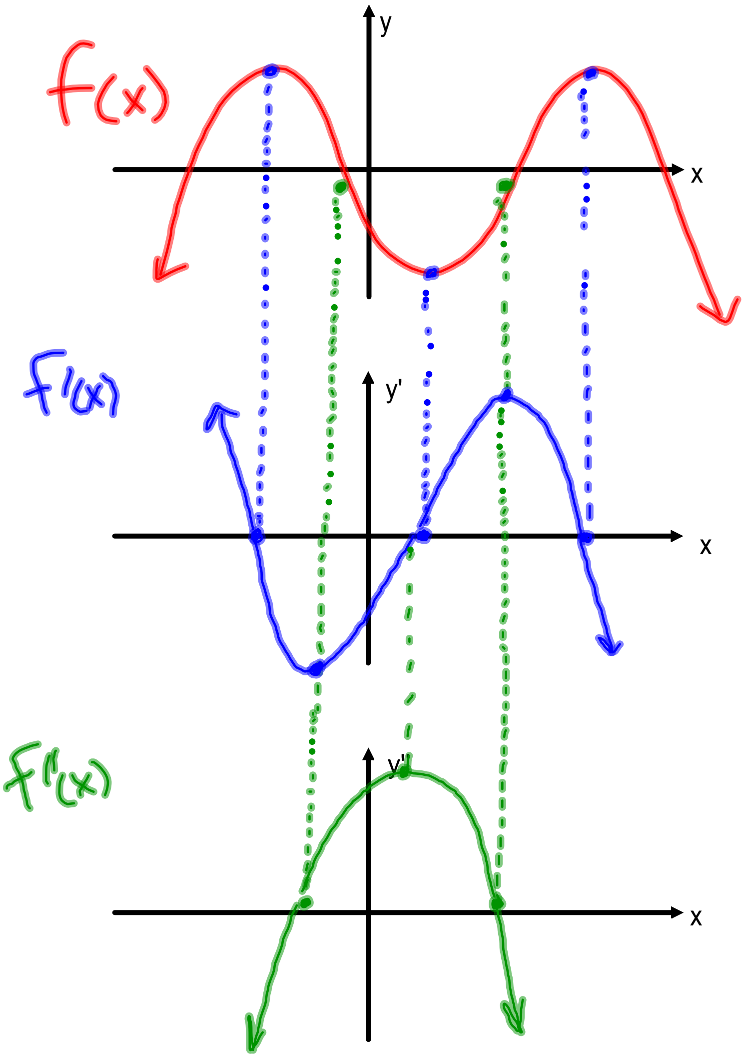
L4. (4.4) Inflection Points, Concavity, & The 2nd Derivative Test



x^4

$4x^3$

$12x^2$

Key Observations:

1- Intervals of INC/DEC are separated by MAX/MIN points.

2- Intervals of CONCAVITY are separated by INFLECTION points.

3- We can find the x-values of inflection points by setting $f''(x) = 0$ and solving.

Inflection Points:

If $f''(a) = 0$, then $(a, f(a))$ is an inflection point.

Concavity:

$f(x)$ is concave up if $f''(x) > 0$.
$f(x)$ is concave down if $f''(x) < 0$.

The 2nd Derivative Test:

At a local max, $f''(x) < 0$.
At a local min, $f''(x) > 0$.

Ex1: Find then classify the critical points of the function
$f(x) = 3x^5 - 25x^3 + 60x$ using the 2nd derivative test.

$f'(x) = 15x^4 - 75x^2 + 60$

$0 = x^4 - 5x^2 + 4$

$= (x^2-4)(x^2-1)$

$= (x-2)(x+2)(x-1)(x+1)$

$x = -2, -1, 1,$ or 2

$f''(x) = 60x^3 - 150x$

$0 = \underline{30x}(\underline{2x^2-5})$

$x = 0$ or $2x^2 - 5 = 0$

$x = \pm\sqrt{\frac{5}{2}}$

	$x<-2$	$-2<x<-1$	$-1<x<1$	$1<x<2$	$x>2$
x^2-4	+	−	−	−	+
x^2-1	+	+	−	+	+
$f'(x)$	+	−	+	−	+

	$x<-\sqrt{\frac{5}{2}}$	$-\sqrt{\frac{5}{2}}<x<0$	$0<x<\sqrt{\frac{5}{2}}$	$x>\sqrt{\frac{5}{2}}$
$30x$	−	−	+	+
$2x^2-5$	+	−	−	+
$f''(x)$	−	+	−	+

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Ex2: Graph $f(x) = x^2 - 2x^4$ by first finding

a) the intercepts
b) the critical points
c) the intervals of INC/DEC
d) the inflection points
e) the intervals of concavity

Note: There is no need to use limits here since we know polys. have no asymptotes.

(a) $f(x) = x^2 - 2x^4$

$0 = -x^2(2x^2 - 1)$

$x = 0, \pm\sqrt{\frac{1}{2}}$

(b) $f'(x) = -8x^3 + 2x$ → FACTOR BEFORE

$f'(x) = -2x(4x^2 - 1)$

$0 = -4x^3 + x$

$0 = -x(4x^2 - 1)$ $\quad f(-\sqrt{\frac{1}{4}}) = 0.125$

$x = 0$ $(0,0)$ or and $\pm\sqrt{\frac{1}{4}}$ $\quad f(\sqrt{\frac{1}{4}}) = 0.125$

	$x < -\sqrt{\frac{1}{4}}$	$-\sqrt{\frac{1}{4}} < x < 0$	$0 < x < \sqrt{\frac{1}{4}}$	$x > \sqrt{\frac{1}{4}}$
$-2x$	+	+	−	−
$4x^2 - 1$	+	−	−	+
$f'(x)$	+	−	+	−

INTERVALS OF INC. $(-\infty, -\sqrt{\frac{1}{4}})$ & $(0, \sqrt{\frac{1}{4}})$.

" " DEC. $(-\sqrt{\frac{1}{4}}, 0)$ & $(\sqrt{\frac{1}{4}}, \infty)$.

d) Find $f''(x) = 0$

$f''(x) = -24x^2 + 2$

$= -2(12x^2 - 1)$

$0 = -2(12x^2 - 1)$

$x = \pm\sqrt{\frac{1}{12}}$

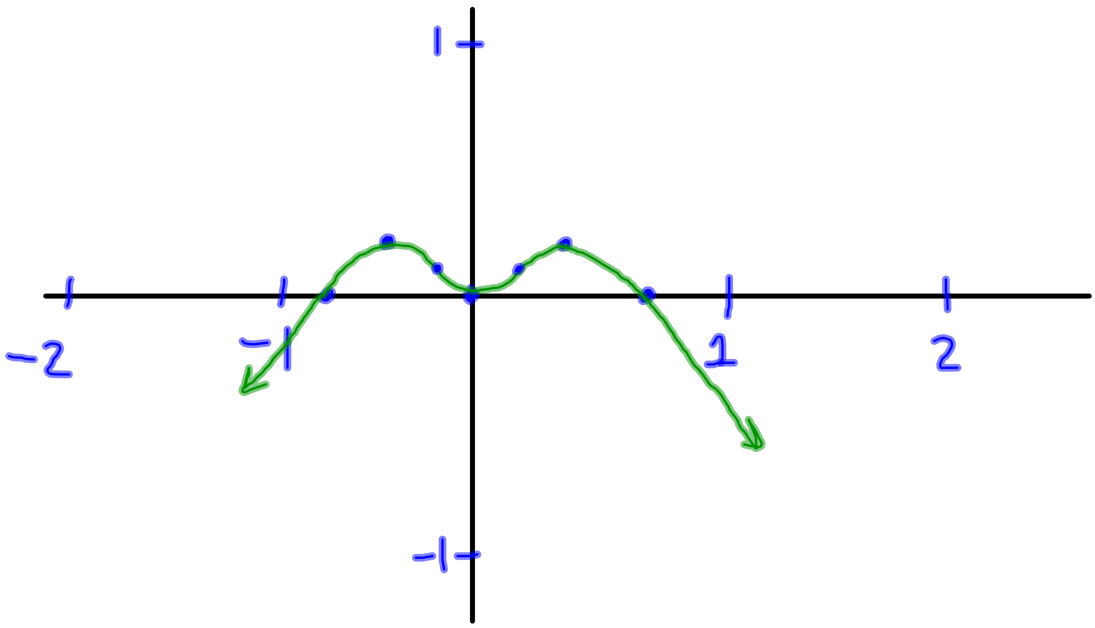
$f(\sqrt{\frac{1}{12}}) = (\sqrt{\frac{1}{12}})^2 - 2(\sqrt{\frac{1}{12}})^4$

$= \frac{1}{12} - \frac{2}{144}$

$= \frac{10}{144}$

$= \frac{5}{72}$ $\quad$ & $f(-\sqrt{\frac{1}{12}}) \doteq 0.07$

$\doteq 0.07$


1
-1
-2
-1
1
2

The Algorithm for Curve Sketching:

- Domain
- Asymptotes
- Intercepts
- Critical Points
- Intervals of INC/DEC
- Inflection Points
- Intervals of Concavity

Assigned Work:

p.205 #2, 3, 5, 13a