L4. (4.4) Inflection Points, Concavity, & The 2nd Derivative Test

\[ f(x) \]

\[ f'(x) \]

\[ f''(x) \]
Key Observations:

1- Intervals of \textbf{INC/DEC} are separated by \textbf{MAX/MIN} points.

2- Intervals of \textbf{CONCAVITY} are separated by \textbf{INFLECTION} points.

3- We can find the x-values of inflection points by setting $f''(x) = 0$ and solving.

\textbf{Inflection Points:}
If $f''(a) = 0$, then $(a, f(a))$ is an inflection point.

\textbf{Concavity:}
- $f(x)$ is concave up if $f''(x) > 0$.
- $f(x)$ is concave down if $f''(x) < 0$.

\textbf{The 2nd Derivative Test:}
- At a local max, $f''(x) < 0$.
- At a local min, $f''(x) > 0$. 
Ex1: Find then classify the critical points of the function  \( f(x) = 3x^5 - 25x^3 + 60x \) using the 2nd derivative test.

\[
\begin{align*}
\frac{f'(x)}{} &= 15x^4 - 75x^2 + 60 \\
0 &= x^4 - 5x^2 + 4 \\
&= (x^2 - 4)(x^2 - 1) \\
&= (x - 2)(x + 2)(x - 1)(x + 1) \\
x &= -2, -1, 1, \text{ or } 2 \\
\frac{f''(x)}{} &= 60x^3 - 150x \\
0 &= 30x(2x^2 - 5) \\
x &= 0 \text{ or } 2x^2 - 5 = 0 \\
x &= \pm \frac{\sqrt{10}}{2}
\end{align*}
\]

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 - 4 )</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( x^2 - 1 )</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>( -\frac{\sqrt{5}}{2} )</th>
<th>0</th>
<th>( \frac{\sqrt{5}}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 - \frac{5}{4} )</td>
<td>( \frac{5}{2} &lt; x &lt; 0 )</td>
<td>( 0 &lt; x &lt; \frac{\sqrt{5}}{2} )</td>
<td>( x &gt; \frac{\sqrt{5}}{2} )</td>
</tr>
<tr>
<td>( x^2 )</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( 2x^2 - 5 )</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( f''(x) )</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

\[UPL\]
Ex2: Graph $f(x) = x^2 - 2x^4$ by first finding

a) the intercepts
b) the critical points
c) the intervals of INC/DEC
d) the inflection points
e) the intervals of concavity

Note: There is no need to use limits here since we know polys. have no asymptotes.

(a) $f(x) = x^2 - 2x^4$

$x = 0, \pm \sqrt{\frac{1}{2}}$

(b) $f'(x) = -8x^3 + 2x$

$x = 0, \pm 0.5$

$c_{0.5} = -2 \times (4x^2 - 1)$

$c_{0.5} = -2 \times (4(0.5)^2 - 1)$

$c_{0.5} = -2 \times (4(0.25) - 1)$

$c_{0.5} = -2 \times (1 - 1)$

$c_{0.5} = 0$

$x = 0, \pm 0.5$

$f(0) = 0$

$f(0.5) = 0.125$

$f(-0.5) = 0.125$

(c) Intervals of INC/DEC:

<table>
<thead>
<tr>
<th>$x$</th>
<th>INC/DEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &lt; 0$</td>
<td>+</td>
</tr>
<tr>
<td>$0 &lt; x &lt; 0.5$</td>
<td>-</td>
</tr>
<tr>
<td>$x &gt; 0.5$</td>
<td>+</td>
</tr>
</tbody>
</table>

Intervals of INC: $(-\infty, 0) \cup (0.5, \infty)$

Intervals of DEC: $(-0.5, 0)$

(d) Find $f''(x) = 0$

$f''(x) = -48x^2 + 2$

$x = 0, \pm \sqrt{\frac{1}{12}}$

$f'(-\sqrt{\frac{1}{12}}) = (\sqrt{\frac{1}{12}})^2 - 2(\sqrt{\frac{1}{12}})^4$

$f'(-\sqrt{\frac{1}{12}}) = \frac{1}{12} - \frac{2}{144}$

$f'(-\sqrt{\frac{1}{12}}) = \frac{10}{192} - \frac{2}{144}$

$f'(-\sqrt{\frac{1}{12}}) = \frac{8}{192} = 0.07$

$f(-\sqrt{\frac{1}{12}}) = 0.07$. 

- $f(-\sqrt{\frac{1}{12}}) = 0.07$
The Algorithm for Curve Sketching:
- Domain
- Asymptotes
- Intercepts
- Critical Points
- Intervals of INC/DEC
- Inflection Points
- Intervals of Concavity

Assigned Work:

p.205 #2, 3, 5, 13a