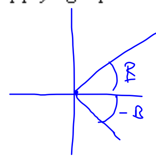
Lesson #4.3: Compound Angle Formulas

A **compound angle** is a trigonometric expression that depends on two or more angles.

Development of the compound angle formula: Compound Angle Proof

Addition Formula for Sine $\boxed{\sin(A+B) = \sin A\cos B + \cos A\sin B}$

Once this one formula has been developed, the others can be developed by applying equivalent trigonometric expressions.



$\cos(-B) = \cos B$

$\sin(-B) = -\sin B$

Subtraction Formula for Sine:

$$\begin{aligned}\sin(A-B) &= \sin(A+(-B))\\ &= \sin A\cos(-B) + \cos A\sin(-B)\\ &= \sin A\cos B + \cos A(-\sin B)\\ &= \sin A\cos B - \cos A\sin B\end{aligned}$$

$\boxed{\sin(A-B) = \sin A\cos B - \cos A\sin B}$

Addition Formula for Cosine

$\sin x = \cos\left(\frac{\pi}{2} - x\right)$

$\cos x = \sin\left(\frac{\pi}{2} - x\right)$

$$\begin{aligned}\cos(x+y) &= \sin\left(\frac{\pi}{2} - (x+y)\right)\\ &= \sin\left(\left(\frac{\pi}{2} - x\right) - y\right)\\ &= \sin\left(\frac{\pi}{2} - x\right)\cos y - \cos\left(\frac{\pi}{2} - x\right)\sin y\\ &= \cos x\cos y - \sin x\sin y\end{aligned}$$

$\boxed{\cos(x+y) = \cos x\cos y - \sin x\sin y}$

Subtraction Formula for Cosine:

$\boxed{\cos(x-y) = \cos x\cos y + \sin x\sin y}$

The following are important trigonometric relationships:

sin(A + B) = sinAcosB + cosAsinB

cos(A + B) = cosAcosB – sinAsinB

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

To find sin(A - B), cos(A - B) and tan(A - B), just change the + signs in the above identities to - signs and vice-versa:

sin(A - B) = sinAcosB – cosAsinB

cos(A - B) = cosAcosB + sinAsinB

$\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Double Angle Formula:

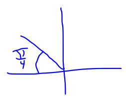
$\sin(2A) = 2\sin A\cos A$

$\tan(2A) = \dfrac{2\tan A}{1-\tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = \cos^2 A - 1$

$\cos 2A = 1 - \sin^2 A$



Example #1: Use an appropriate compound angle formula to express as a single trigonometric function, and then determine an exact value for each.

a) $\sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6} = \sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)$

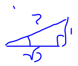
$A = \frac{\pi}{3} \quad B = \frac{\pi}{6}$

$= \sin\left(\frac{\pi}{2}\right)$

$= 1$

b) $\cos\frac{\pi}{3}\cos\frac{5\pi}{12} - \sin\frac{\pi}{3}\sin\frac{5\pi}{12} = \cos\left(\frac{\pi}{3}+\frac{5\pi}{12}\right)$

$= \cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$



c) $\sin\frac{5\pi}{9}\cos\frac{7\pi}{18} - \cos\frac{5\pi}{9}\sin\frac{7\pi}{18}$

$= \sin\left(\frac{5\pi}{9} - \frac{7\pi}{18}\right)$

$= \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

d) $\cos\frac{5\pi}{12}\cos\frac{\pi}{4} + \sin\frac{5\pi}{12}\sin\frac{\pi}{4}$

$= \cos\left(\frac{5\pi}{12} - \frac{\pi}{4}\right)$

$= \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Example #2: Apply a compound angle formula, and then determine an exact value for each.

$\cos\left(\frac{11\pi}{12}\right)$ **a)** $\cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$

$= \cos\frac{3\pi}{4}\cos\frac{\pi}{6} + \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$

$= -\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2}$

$= \frac{-\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$= \frac{\sqrt{2}-\sqrt{6}}{4}$

b) $\cancel{\sin\left(\frac{5\pi}{4} - \frac{2\pi}{3}\right)}$

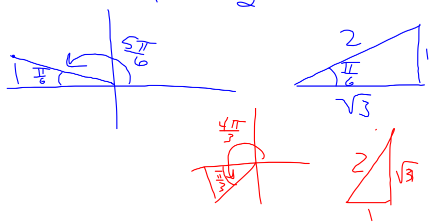
c) $\cos\left(\frac{5\pi}{6} + \frac{4\pi}{3}\right)$

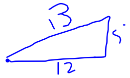
$= \cos\frac{5\pi}{6}\cos\frac{4\pi}{3} - \sin\frac{5\pi}{6}\sin\frac{4\pi}{3}$

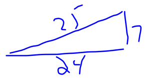
$= -\frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right) - \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$







Example #3: Angle x is in the first quadrant and angle y is in the second quadrant such that $\cos x = \frac{12}{13}$, and $\sin y = \frac{7}{25}$. Determine an exact value for:

a) $\sin x = \frac{5}{13}$

b) $\cos y = \frac{24}{25}$

c) $\sin(x + y)$

$= \sin x \cos y + \cos x \sin y$

$= \frac{5}{13}\left(\frac{24}{25}\right) + \frac{12}{13}\left(\frac{7}{25}\right)$

$= \frac{120}{325} + \frac{84}{325}$

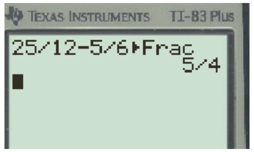
$= \frac{204}{325}$

d) $\cos(x - y)$

e) $\tan(x + y)$

4. Use an appropriate compound angle formula to determine an exact value for each.

a) $\sin \frac{11\pi}{12}$



b) $\cos \frac{25\pi}{12}$

$= \cos\left(\frac{5\pi}{6} + \frac{5\pi}{4}\right)$

$= \cos\frac{5\pi}{6}\cos\frac{5\pi}{4} - \sin\frac{5\pi}{6}\sin\frac{5\pi}{4}$

$= -\frac{\sqrt{3}}{2}\left(-\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

5. Angle b lies in the second quadrant such that $\cos b = -\frac{3}{5}$.

a) Determine an exact answer for $\sin b$ and $\tan b$.

b) Determine an exact answer for $\cos 2b$.

c) Determine an exact answer for $\sin 2b$.

d) Determine an exact answer for $\tan 2b$.

e) Use a calculator to determine an approximate measure for b, in radians, to two decimal places.

f) In which quadrant does angle $2b$ lie? Justify your answer.

Homework: pg. 232; 1 – 11, 15

Compound_Angle_Proof[1].ppt