#4

$$\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$

$$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

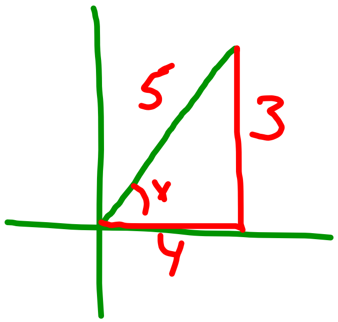
$$* \quad \frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$$

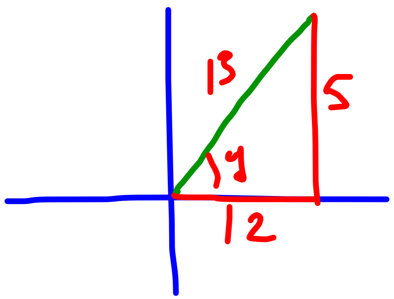
$$\sin(x+y)$$

$$= \sin x\cos y + \cos x\sin y$$

#8.

$\sin x = \frac{3}{5}$ $\qquad$ $\cos y = \frac{5}{13}$



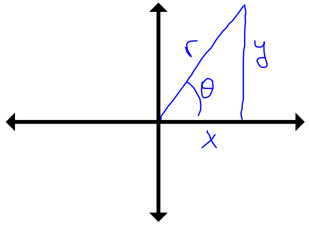


a) $\cos x = \frac{4}{5}$ $\qquad$ b) $\sin y = \frac{12}{13}$

Lesson #4.5: Proving Trigonometric Identities

The basic trigonometric identities are the Pythagorean identity, the quotient identity, the reciprocal identities and the compound angle formulas.
You can use these identities to prove more complex identities.

Pythagorean Identity



$x^2 + y^2 = r^2$

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$

$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

Quotient Identity

$\tan\theta = \frac{y}{x}$

$= \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{\sin\theta}{\cos\theta}$

$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$

Reciprocal Identities:

$\csc\theta = \frac{1}{\sin\theta}$
$\sec\theta = \frac{1}{\cos\theta}$
$\cot\theta = \frac{1}{\tan\theta}$

Compound Angle Formulas

$\sin(x+y) = \sin x\cos y + \cos x\sin y$
$\sin(x-y) = \sin x\cos y - \cos x\sin y$
$\cos(x+y) = \cos x\cos y - \sin x\sin y$
$\cos(x-y) = \cos x\cos y + \sin x\sin y$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

$(a+b)^2 = a^2 + 2ab + b^2$

Example #1: Simplify each expression:

a) $\cos x \csc x \tan x$

$= \cancel{\cos x} \cdot \dfrac{1}{\cancel{\sin x}} \cdot \dfrac{\cancel{\sin x}}{\cancel{\cos x}}$

$= 1$

b) $\cos x \cot x + \sin x$

$= \cos x \dfrac{\cos x}{\sin x} + \sin x \cdot \dfrac{\sin x}{\sin x}$

$= \dfrac{\cos^2 x}{\sin x} + \dfrac{\sin^2 x}{\sin x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\sin x} = \dfrac{1}{\sin x} = \csc x$

c) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$

$= \sin^2 x + \cancel{2\sin x\cos x} + \cos^2 x + \sin^2 x \cancel{- 2\sin x\cos x} + \cos^2 x$

$= 1 + 1$

$= 2$

Example #2: Prove that:

a) $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

$LS = \sin\left(x + \frac{\pi}{2}\right)$ $\quad RS = \cos x$

$= \sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2}$

$= \sin x(0) + \cos x(1)$

$= \cos x$

$\therefore LS = RS$

$\therefore \sin\left(x + \frac{\pi}{2}\right) = \cos x$

b) $\cos\left(\frac{3\pi}{4} - x\right) - \sin\left(\frac{3\pi}{4} + x\right) = \cancel{-}\sqrt{2}(\cos x \cancel{+} \sin x)$

$\boxed{\dfrac{2}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{\sqrt{2}}\right) = \dfrac{2\sqrt{2}}{2} = \sqrt{2}}$

$LS = \cos\left(\frac{3\pi}{4} - x\right) - \sin\left(\frac{3\pi}{4} + x\right)$

$= \cos\frac{3\pi}{4}\cos x + \sin\frac{3\pi}{4}\sin x - \left(\sin\frac{3\pi}{4}\cos x + \cos\frac{3\pi}{4}\sin x\right)$

$= -\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x - \left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)$

$= -\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x$

$= -\frac{1}{\sqrt{2}}(\cos x - \sin x + \cos x - \sin x)$

$= -\frac{1}{\sqrt{2}}(2\cos x - 2\sin x) = -\frac{2}{\sqrt{2}}(\cos x - \sin x)$

$= \sqrt{2}(\cos x - \sin x)$

$= RS.$

c) $\dfrac{\sin 2x}{1 - \cos 2x} = \cot x$

$LS = \dfrac{\sin 2x}{1 - \cos 2x}$ $\quad RS = \dfrac{\cos x}{\sin x}$

$= \dfrac{2\sin x\cos x}{1 - (1 - 2\sin^2 x)}$

$= \dfrac{2\sin x \cos x}{1 - 1 + 2\sin^2 x}$

$= \dfrac{2\sin x\cos x}{2\sin^2 x}$

$= \dfrac{\cos x}{\sin x}$

$\therefore LS = RS.$

d) Use graphing technology to determine whether it is reasonable to conjecture that $\sin^4 x - \cos^4 x = 2\sin^2 x + 1$ is an identity. If it appears to be an identity, prove the identity. If not, determine a counterexample.

NOT an identity.

Let $x = 0$

$LS = \sin^4(0) - \cos^4(0)$

$= -1$

$RS = 2\sin^2(0) + 1$

$= 1$

$LS \neq RS.$

Homework: pg. 240; 1 – 13, 15 – 18

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