Assessment Quiz #4 of Unit 7

Find the vector equation of the plane that contains the points P(1, -2, 5) and Q(3, 1, 2) and is parallel to the line

$\vec{r} = (0,3,1) + t(2,4,1)$

Q

P

$\vec{r} = (1,-2,5) + s(2,4,1) + u(2,3,-3)$

$, s, u \in \mathbb{R}$

L5(8.5) Cartesian Equation (A.K.A Scalar Eq^n) of a Plane

Investigation

Consider the following equation of a plane:

$$\vec{r} = (1,2,5) + s(5,2,3) + t(6,3,-5)$$

A normal to this plane would be:

5 2 3 5 2 3
6 3 -5 6 3 -5

$$\vec{n} = (5,2,3) \times (6,3,-5)$$
$$= (-19, 43, 3) \leftarrow A, B, C$$

The dot product between $\vec{n}$ and any vector in the plane $\vec{r}$ will equal zero.

$$\vec{n} \bullet \vec{r} = 0$$

or

$$\vec{n} \bullet \overrightarrow{P_0P} = 0$$

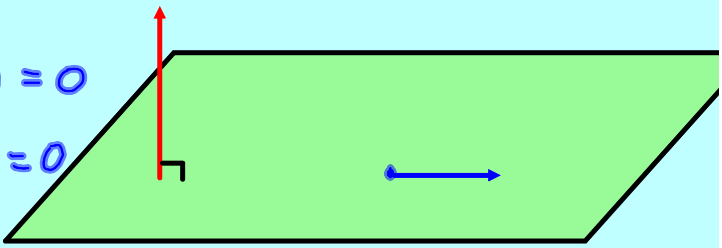
where $P_0 = (1,2,5)$
$P = (x,y,z)$
$\overrightarrow{P_0P} = (x-1,y-2,z-5)$

$$(-19,43,3) \bullet (x-1,y-2,z-5) = 0$$

$$-19(x-1) + 43(y-2) + 3(z-5) = 0$$

$$-19x + 19 + 43y - 86 + 3z - 15 = 0$$

$$\boxed{-19x + 43y + 3z - 82 = 0}$$



The Cartesian Equation of a plane in 3D is defined by:

$$Ax + By + Cz + D = 0$$

Where (A,B,C) is normal to the plane.

Ex1: Find the Cartesian equation of the plane that contains the point (2, 5, 1) and is perpendicular to the line

$$\vec{r} = (0,3,1) + t(2,4,1)$$

$\vec{n} = (2,4,1)$

Sub in $\vec{n}$ into $Ax + By + Cz + D = 0$;

$2x + 4y + z + D = 0$

Sub in $(2,5,1)$ to find constant D;

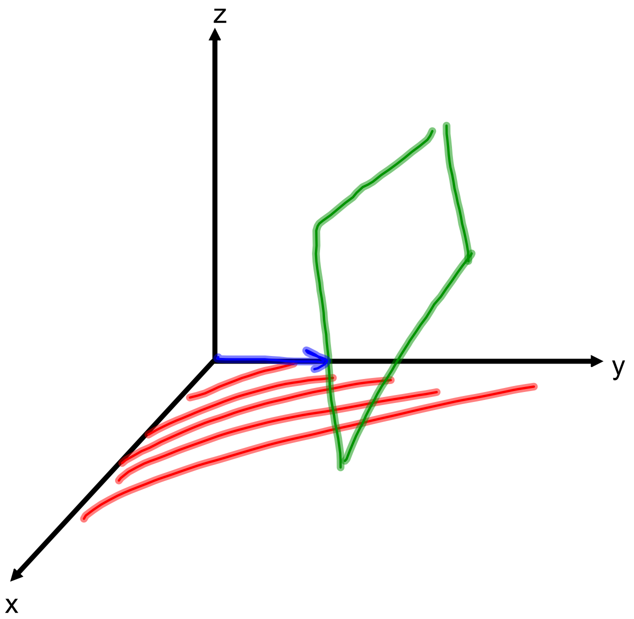
$2(2) + 4(5) + (1) + D = 0$

$D = -25$

$\therefore$ The Cartesian equation of the plane is $2x + 4y + z - 25 = 0$.

Ex2: Find a Cartesian equation of any plane perpendicular to the xy plane.

$(z=0)$



$\hat{j}=(0,1,0)$ sub in ;

$Ax+By+Cz+D=0$

$A(0)+B(1)+C(0)+D=0$

D is arbitrarily chosen i.e. $D=0$

$\pi: y=0$

What do you notice about the values of A and B?

Ex3: Find the scalar equation of the plane defined by:

$$x = 2 - q + p$$
$$y = 3 + 4q - p$$
$$z = p - q$$

Vector equation: $\pi: \ \vec{r} = (2,3,0) + q(-1,4,-1) + p(1,-1,1)$

$$Ax + By + Cz + D = 0$$

$$\vec{n} = (-1,4,-1) \times (1,-1,1)$$

$$= \begin{vmatrix} i & j & k \\ -1 & 4 & -1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= (3, 0, -3)$$

Sub $\vec{n}$ in for A, B, C.

What do you notice about the values of A and B?

$$3x - 3z + D = 0$$

Next,

Sub $(2,3,0)$ in to get D

$$3(2) - 3(0) + D = 0$$

$$D = -6$$

$\therefore$ The Cartesian eq'n is

$$3x - 3z - 6 = 0$$

Ex4: State three points on the plane $2x - 5y + z + 4 = 0$.

Choose x & y, solve for z:

$x=0, y=0$

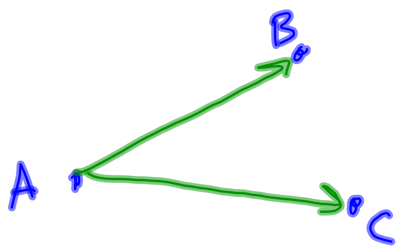
$z=-4$

$A(0,0,-4)$

similarly for other points

$B(-2,0,0)$ $C(1,1,-1)$

How would we change the Cartesian equation of a plane into a vector equation?



$\vec{AB} = (-2,0,4)$

$\vec{AC} = (1,1,3)$

$$\vec{r} = (0,0,-4) + s(-2,0,4) + t(1,1,3), \quad t,s \in \mathbb{R}$$

Assigned Work:

p.468-469 #1, 2, 3, 4b, 6,
8,10, 11, 13, 14

Homework
p. 460
#9

$\pi: \vec{r} = (4,1,6) + s(11,-1,3) + t(-7,2,-2)$

$L: \vec{r} = u(0,0,1)$ Now see example

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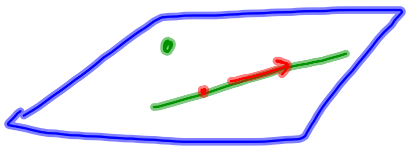

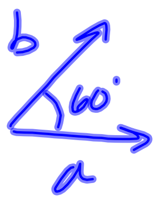

$|\vec{a}\cdot\vec{b}| = |1||1|\cos 60°$

$= \frac{1}{2}$

$(\vec{a} + 3\vec{b}) \cdot (k\vec{a} - \vec{b}) = 0$

$k\vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + 3k\vec{a}\cdot\vec{b} - 3\vec{b}\cdot\vec{b} = 0$

$k - \frac{1}{2} + \frac{3k}{2} - 3 = 0$

$\frac{5k}{2} = \frac{7}{2}$

$\therefore k = \frac{7}{5}$