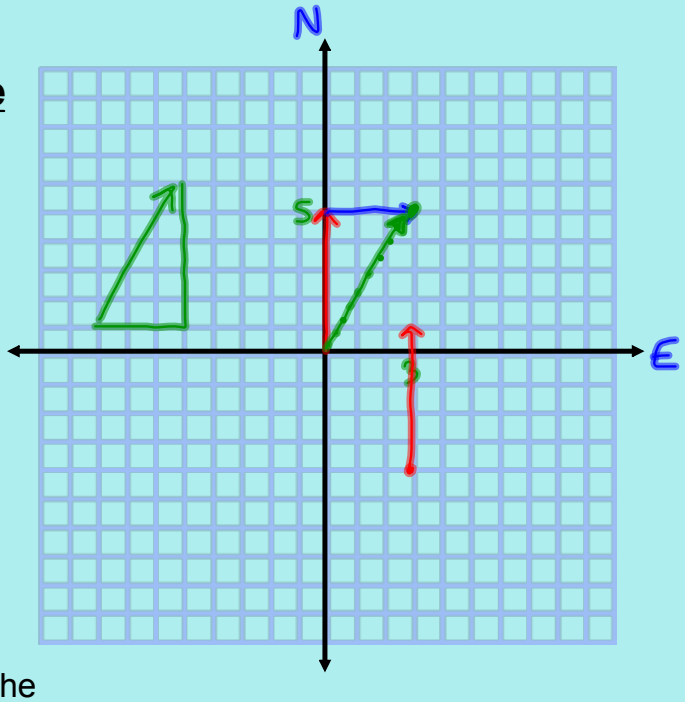
L5 (6.5) Algebraic Vectors in $\mathbf{R^2}$ and $\mathbf{R^3}$

Vectors in 2-Dimensional Space

Draw the following vectors in two dimensions.
5km/h north
3km/h east



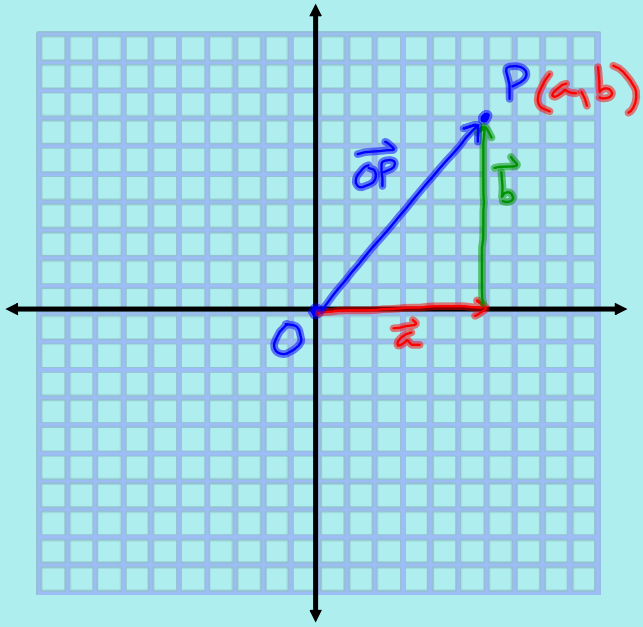
What are their coordinates? (3,5)

Could you draw the same vector with different co-ordinates? Y

If I asked you to draw 5km/h north with the origin of (3,-4) how would it differ? STARTS AT DIFF SPOT.

Vectors in 2-Dimensional Space

If we choose any point P (a,b), how can we determine the vector from the origin to P?



$\vec{OP}$ is the resultant vector of $\vec{a} + \vec{b}$

What are the magnitude and direction for $\vec{OP}$?

$$|\vec{OP}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2}$$

In Summary:

- Any point P(x, y), in $\mathbf{R}^2$, can be thought of as the vector $\overrightarrow{OP}$ = (x, y).
- The direction of the vector $\overrightarrow{OP}$ = (x, y) is implied by the coordinate.
- The magnitude of OP can be calculated using

$$\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2}$$

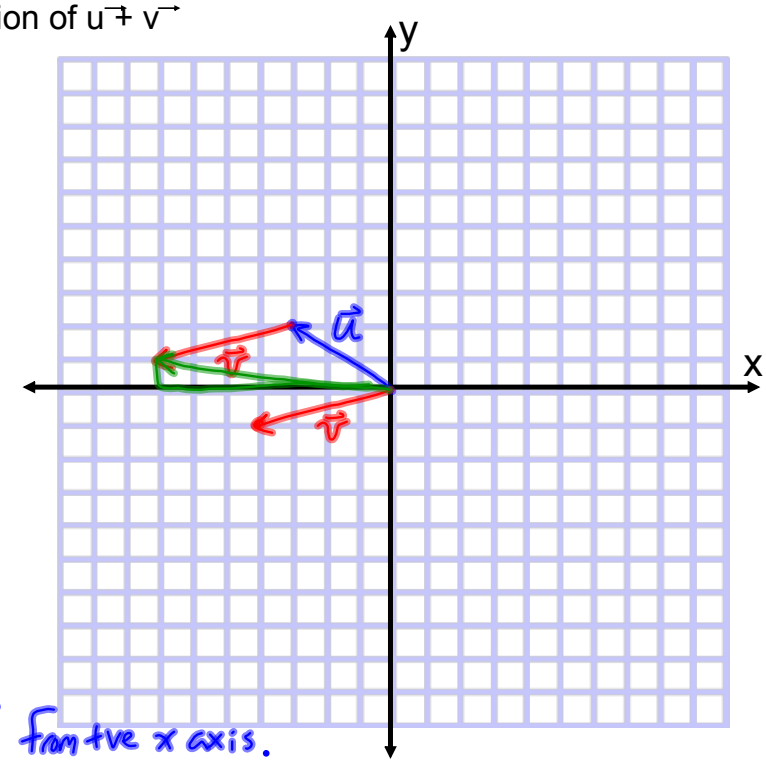
Note: The vector $\overrightarrow{OP}$ = (x, y) is know as the position vector of the point P(x, y)

Ex1: Given vector $\vec{u} = (-3,2)$ and $\vec{v} = (-4,-1)$

a) Sketch $\vec{u}$, $\vec{v}$ and $\vec{u} + \vec{v}$ showing triangle addition

b) Find the magnitude and direction of $\vec{u} + \vec{v}$

c) Find $\vec{u} + \vec{v}$ in algebraic form



(b) $|\vec{u}+\vec{v}| = \sqrt{1^2+7^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$

$\doteq 7.07$

$\theta = \tan^{-1}\left(\frac{1}{7}\right)$

$\doteq 8^\circ$

$\alpha = 180^\circ - 8^\circ = 172^\circ$ from +ve x axis.

(c) $\vec{u} = -3\vec{i} + 2\vec{j}$

$\vec{v} = -4\vec{i} - \vec{j}$

$\vec{u}+\vec{v} = -3\vec{i} + 2\vec{j} - 4\vec{i} - \vec{j}$

$= -7\vec{i} + \vec{j}$

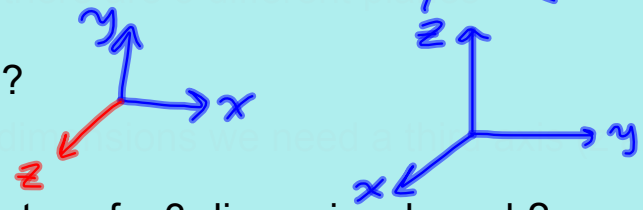
$\vec{u}+\vec{v} = (-7, 1)$

Vectors in 3-Dimensional Space

What is 3-D?

3 Dimensional Space

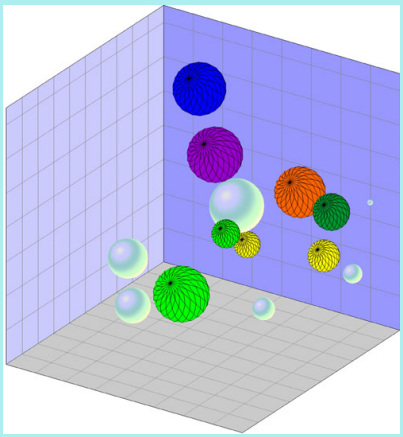
How do we graph 3-D?

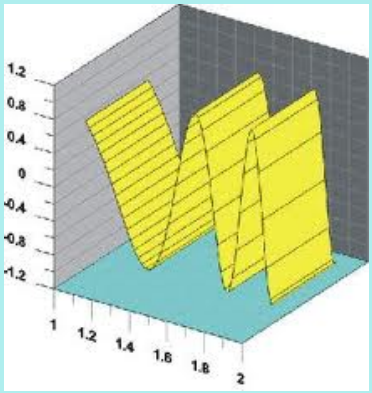


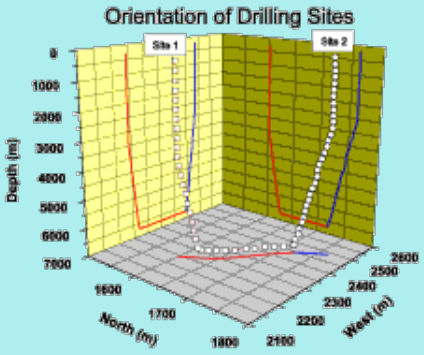
What are the co-ordinates of a 3-dimensional graph?

$(x, y, z) \Rightarrow (\vec{i}, \vec{j}, \vec{k})$

What does a three dimensional grid look like?

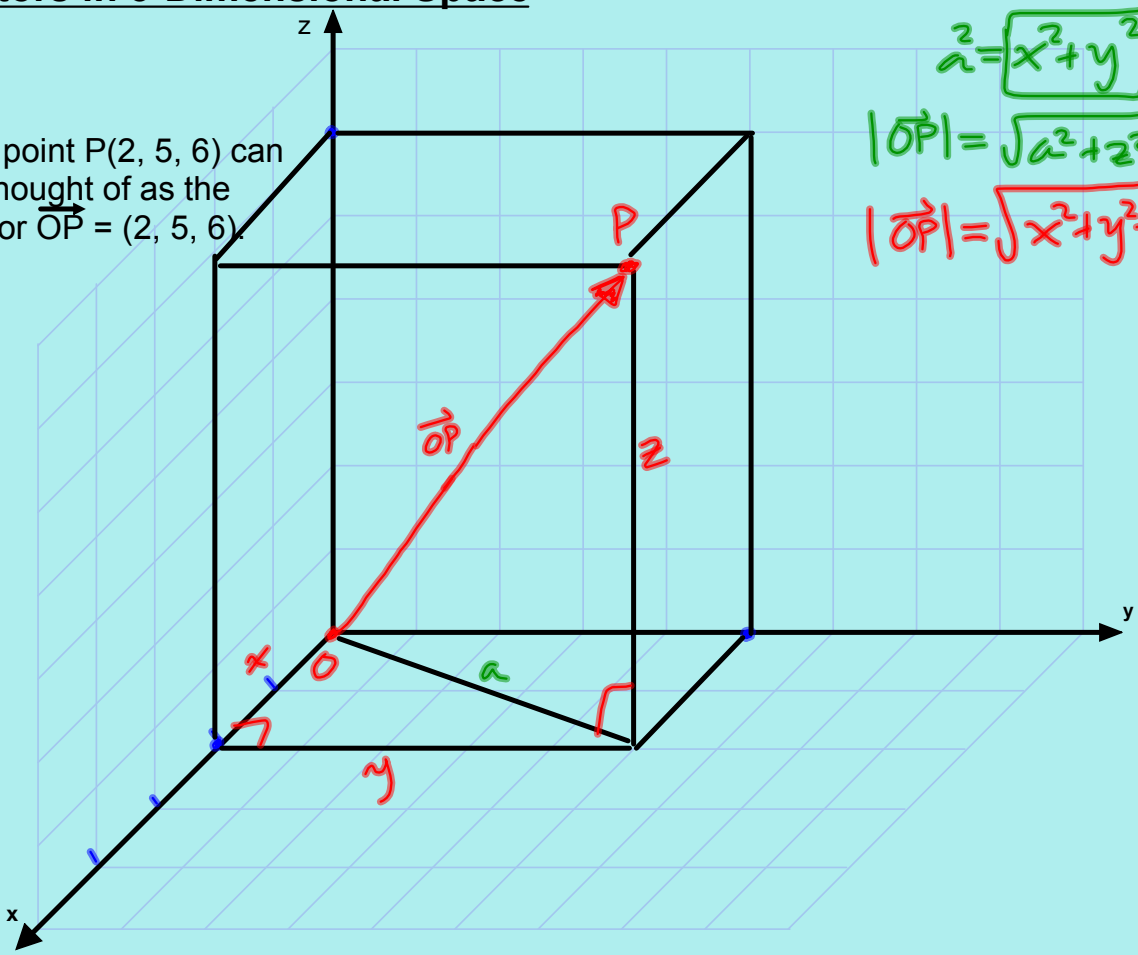






Vectors in 3-Dimensional Space

The point P(2, 5, 6) can be thought of as the vector $\overrightarrow{OP}$ = (2, 5, 6).



$a^2 = x^2 + y^2$

$|\overrightarrow{OP}| = \sqrt{a^2 + z^2}$

$|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$

In Summary:

- Any point P(x, y, z), in $\mathbf{R}^3$, can be thought of as the vector $\overrightarrow{OP}$ = (x, y, z).
- The direction of $\overrightarrow{OP}$ is implied by the coordinates.
- The magnitude of $\overrightarrow{OP}$ can be calculated using

$$\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2 + z^2}$$

Note: The vector $\overrightarrow{OP}$ = (x, y, z) is know as the position vector of the point P(x, y, z)

Where are the following vectors in comparison to our class?

$\vec{a}$ = (1, 2, 3)

$\vec{b}$ = (-2, 3, 4)

$\vec{c}$ = (3, -4, 5)

$\vec{d}$ = (0, 5, -6)

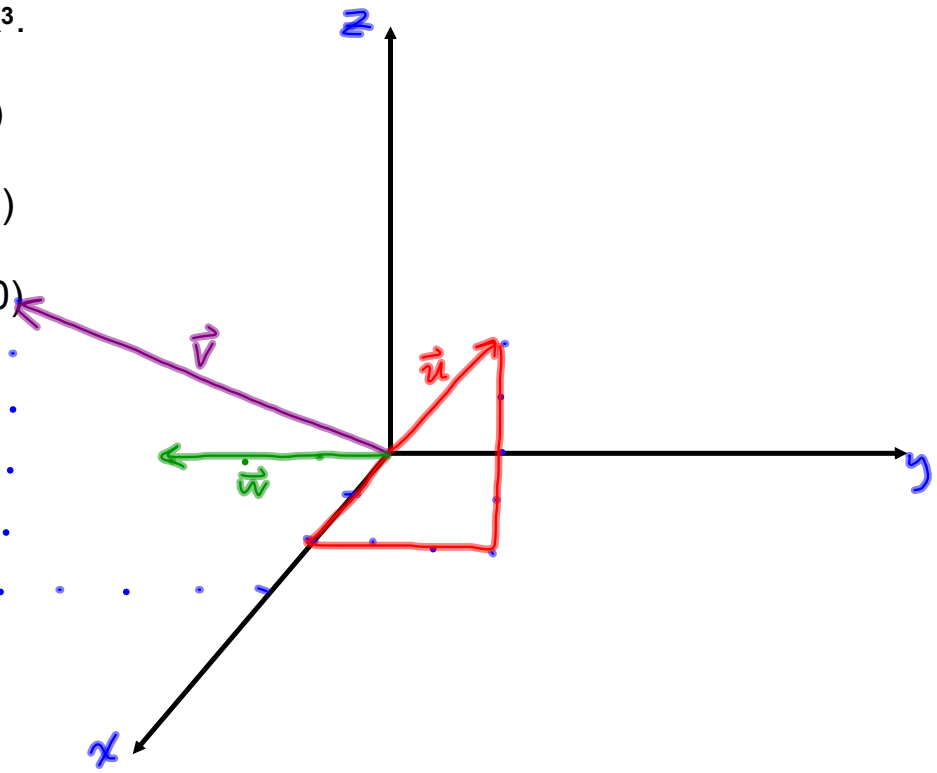
$\vec{e}$ = (0, -3, 0)

Ex2: Draw these vectors within a corresponding rectangular prism (if needed) in $\mathbf{R}^3$.

$\vec{u} = (2, 3, 4)$

$\vec{v} = (3, -4, 5)$

$\vec{w} = (0, -3, 0)$



Assigned work:

p. 316-318 #1, 4, 5, 6, 7ab, 8, 12a, 15