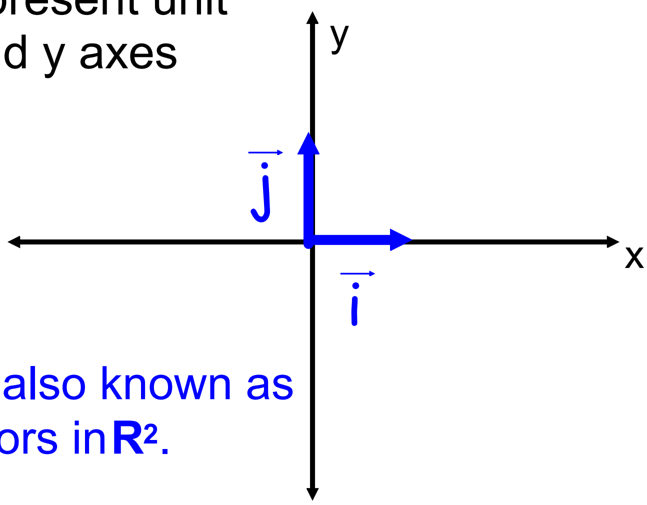
L6 (6.6) Operations with Algebraic Vectors in $\mathbf{R}^2$

Unit Vectors in the direction of the x and y axes

The vectors $\vec{i}$ and $\vec{j}$ represent unit vectors along the x and y axes respectively.

$\vec{i} = (1, 0)$
$\vec{j} = (0, 1)$

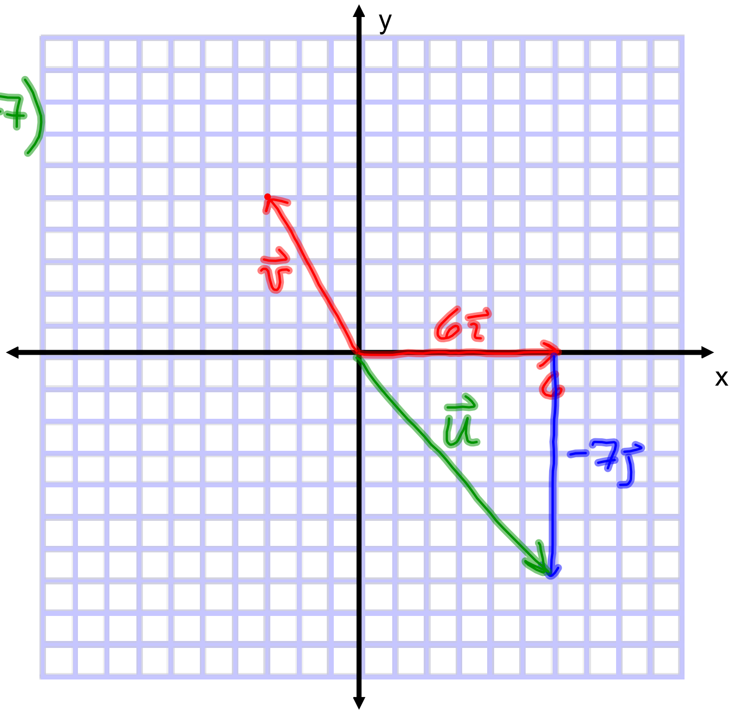


The vectors $\vec{i}$ and $\vec{j}$ are also known as the *standard basis* vectors in $\mathbf{R}^2$.

Ex1: Draw the following vectors in two dimensions.

$\vec{u} = 6\vec{i} - 7\vec{j}$ $= (6,-7)$

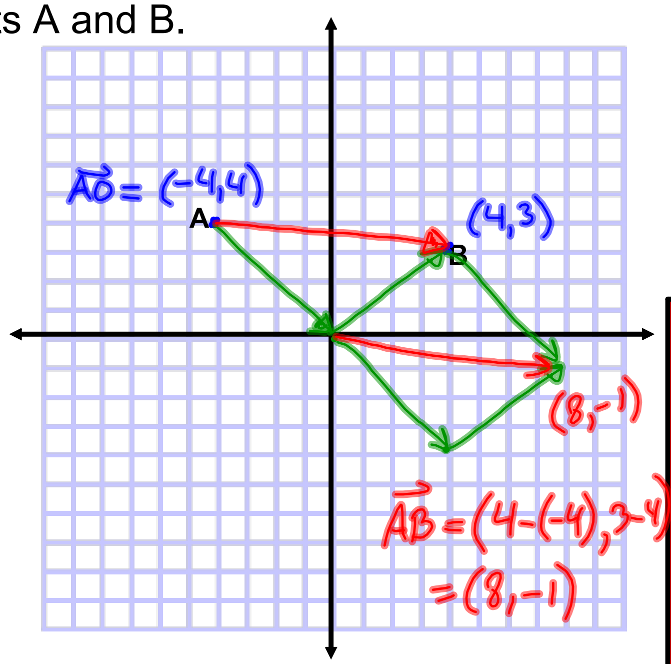
$\vec{v} = -3\vec{i} + 5\vec{j}$



Algebraic Vectors Between Two Points

Given points A (x_1,y_1) and B (x_2,y_2), the vector $\vec{AB}$ can be calculated using the position vectors of points A and B.

$$
\begin{aligned}
\vec{AB} &= \vec{AO} + \vec{OB} \\
&= -\vec{OA} + \vec{OB} \\
\vec{AB} &= \vec{OB} - \vec{OA} \\
&= (x_2,y_2) - (x_1,y_1) \\
\left(\vec{AB}\right. &= \left.(x_2-x_1,y_2-y_1)\right)
\end{aligned}
$$

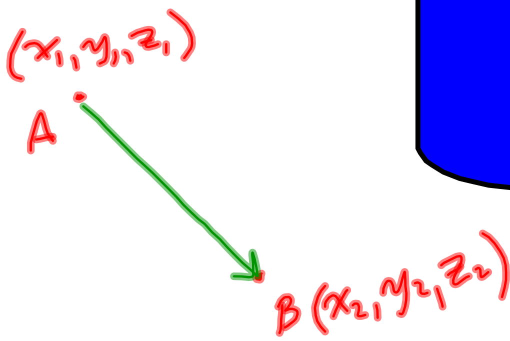


WOOOHOOO! We now have a formula for finding the algebraic vector between two given points.

$$\vec{AB} = \vec{OB} - \vec{OA}$$

NOTE: This can be extended to 3D vectors as well:

$$
\begin{aligned}
\vec{AB} &= \vec{OB} - \vec{OA} \\
&= (x_2,y_2,z_2) - (x_1,y_1,z_1) \\
&= (x_2-x_1,y_2-y_1,z_2-z_1)
\end{aligned}
$$



Ex2: Given the points L(4,-2) and M(-2,3) find:
a) $\overrightarrow{LM}$ and $|\overrightarrow{LM}|$
b) a unit vector in the direction $\overrightarrow{LM}$

(a) $\overrightarrow{LM} = \overrightarrow{OM} - \overrightarrow{OL}$

$= (x_2 - x_1, y_2 - y_1)$

$= (-2-4, 3-(-2))$

$= (-6, 5)$

$|\overrightarrow{LM}| = \sqrt{(-6)^2 + (5)^2}$

$= \sqrt{36+25}$

$= \sqrt{61}$

(b) $\hat{LM} = \left(\frac{-6}{\sqrt{61}}, \frac{5}{\sqrt{61}}\right)$

$= \left(\frac{-6\sqrt{61}}{61}, \frac{5\sqrt{61}}{61}\right)$

Ex3: Given $\vec{u} = \vec{i} - 5\vec{j}$ and $\vec{v} = 4\vec{i} - 10\vec{j}$,
find $|\vec{u} - 2\vec{v}|$.

$\vec{u} = (1, -5)$

$$|\vec{u} - 2\vec{v}| = |\vec{i} - 5\vec{j} - 2(4\vec{i} - 10\vec{j})|$$

$$= |-7\vec{i} + 15\vec{j}|$$

$$= \sqrt{|-7\vec{i}|^2 + |15\vec{j}|^2}$$

$$= \sqrt{49 + 225}$$

$$= \sqrt{274}$$

Assigned Work:

p.325-326 #3, 4, 5b, 6ac, 7c, 8c,
9, 10, 11, 12, 13a

Example 2 p. 563

Determine $\frac{dy}{dx}$ for $\underline{2xy} - \underline{y^3} = \underline{4}$

$$\frac{d}{dx}(2xy) - \frac{d}{dx}(y^3) = \frac{d}{dx}(4)$$

$$2y + 2x\frac{dy}{dx} \quad -3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2x - 3y^2) = -2y$$

$$\frac{dy}{dx} = \frac{-2y}{(2x-3y^2)}$$

$$\sin x + 2\cos(2y) = 1$$

$$2\cos(2y) = 1 - \sin x$$

$\cos^{-1}(x) \neq (\cos x)^{-1}$

$$\cos(2y) = \frac{1}{2} - \frac{1}{2}\sin x$$

$$2y = \cos^{-1}\left(\frac{1}{2} - \frac{1}{2}\sin x\right)$$

$$y = \frac{\cos^{-1}\left(\frac{1}{2} - \frac{1}{2}\sin x\right)}{2}$$

Do it this way insted (implicit differentiation)

$$\sin x + 2\cos(2y) = 1$$

$$\cos x - 2\sin(2y)\,2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\cos x}{-4\sin(2y)}$$