

L6 - Special Case Critical Points

From Lesson 2 (4.2)

If $f'(a) = 0$, then $x = a$ is a critical value and $(a, f(a))$ is a critical point.

or $f'(a)$ is undefined

A critical point can be

(Maybe not all found)

- a maximum
- a minimum
- a point of inflection

CRITICAL POINTS - PART 2

If $f(x)$ has a domain of $x \in \mathbb{R}$ (no restrictions), but $f'(x)$ yields restrictions on x , then these undefined x -values of $f'(x)$ are critical.

$$f'(x) = \frac{4}{3}x(x^2-4)^{-1/3}$$

$$= \frac{4x}{3(x^2-4)^{1/3}}$$

$$f''(x) = \frac{4(3(x^2-4)^{1/3}) - (x^2-4)^{2/3}(2x)(4x)}{(3(x^2-4)^{1/3})^2}$$

$$= \frac{12(x^2-4)^{1/3} - 8x^2(x^2-4)^{-2/3}}{9(x^2-4)^{2/3}}$$

$$= \frac{4(x^2-4)^{-2/3} [3(x^2-4) - 2x^2]}{9(x^2-4)^{2/3}}$$

$$= \frac{4[3x^2-12-2x^2]}{9(x^2-4)^{4/3}}$$

$$= \frac{4x^2-48}{9(x^2-4)^{4/3}}$$

Ex: Graph $f(x) = \sqrt[3]{(x^2 - 4)^2}$ using the curve sketching algorithm.

$$f(x) = (x^2 - 4)^{\frac{2}{3}}$$

a) Check Domain $x \in \mathbb{R}$

$$f'(x) = \frac{2}{3}(x^2 - 4)^{-\frac{1}{3}}(2x) \quad f(0) = (-4)^{\frac{2}{3}}$$

$$= \frac{4x}{3(x^2 - 4)^{\frac{1}{3}}} \quad = 2.5$$

$$f'(x) = 0 \text{ when:}$$

$$0 = 4x$$

$$x = 0$$

$$f'(x) \text{ undefined when: } f(2) = 0 \quad f(-2) = 0$$

$$3(x^2 - 4)^{\frac{1}{3}} = 0$$

$$x^2 - 4 = 0$$

$$x = \pm 2$$

Intervals	$x < -2$	$-2 < x < 0$	$0 < x < 2$	$x > 2$
$4x$	-	-	+	+
$3(x^2 - 4)^{\frac{1}{3}}$	+	-	-	+
$f'(x)$	-	+	-	+

$$f'(x) = \frac{4}{3}x(x^2 - 4)^{-\frac{1}{3}}$$

$$f''(x) = \frac{4}{3}(x^2 - 4)^{-\frac{1}{3}} - \frac{4}{9}x(x^2 - 4)^{-\frac{4}{3}}(2x)$$

$$= \frac{4}{3(x^2 - 4)^{\frac{1}{3}}} \cdot \frac{3(x^2 - 4)^{\frac{2}{3}}}{3(x^2 - 4)^{\frac{2}{3}}} - \frac{8x^2}{9(x^2 - 4)^{\frac{4}{3}}}$$

$$= \frac{12(x^2 - 4) - 8x^2}{9(x^2 - 4)^{\frac{4}{3}}}$$

$$= \frac{12x^2 - 48 - 8x^2}{9(x^2 - 4)^{\frac{4}{3}}}$$

$$= \frac{4x^2 - 48}{9(x^2 - 4)^{\frac{4}{3}}}$$

Find inflection pts wh $f''(x) = 0$

$$0 = 4x^2 - 48$$

$$x = \pm\sqrt{12}$$

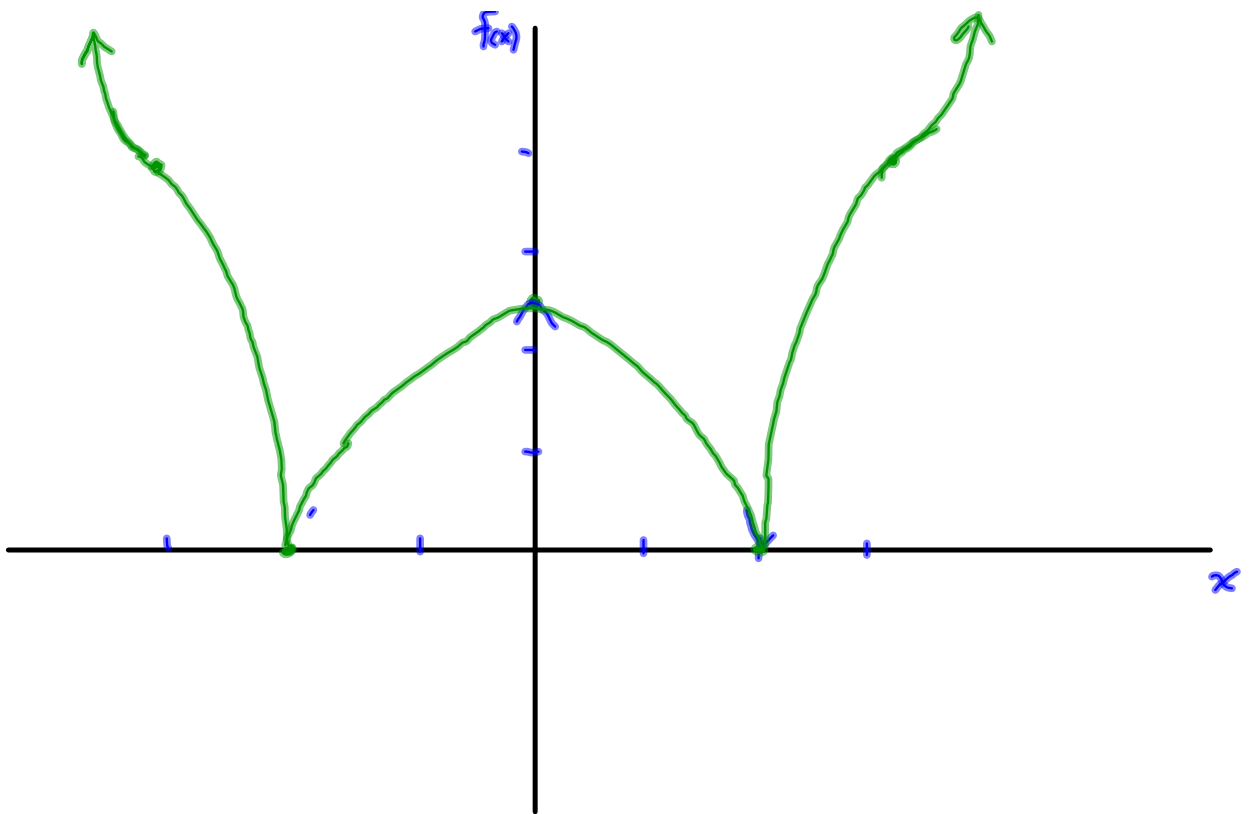
$$x = \pm 2\sqrt{3}$$

$$x = \pm 3.46$$

$$f(3.46) = 4$$

$$f(-3.46) = 4$$

Intervals	$x < -3.46$	$-3.46 < x < -2$	$-2 < x < 2$	$2 < x < 3.46$	$x > 3.46$
$4x^2 - 48$	+	-	-	-	+
$9(x^2 - 4)^{2/3}$	+	+	+	+	+
$f''(x)$	+	-	-	-	+



Next Two Periods:

p. 212 # 4fghij

UNIT 2 REVIEW

p. 216 # 1, 2, 3, 4, 5,
8, 10, 19, 20

