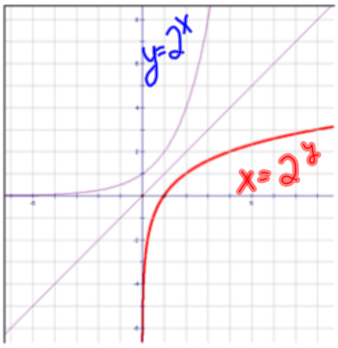
Lesson #6.2: Logarithms

In Lesson #6.1, we look at exponential functions and the inverse of these functions.

Because each x-value of the inverse graph gave a unique y-value, the inverse function is also a function of x

If $y = 2^x$, then the inverse function could be written as $x = 2^y$

and in general,
if $y = b^x$, then the inverse function could be expressed as $x = b^y$



In the study of functions, mathematicians usually prefer to express the y-coordinate in terms of the x-coordinate.
So we restate the relationship as "**y equals the logarithm of x to the base b**" or $\mathbf{y = \log_b x}$

$2^3 = 8 \leftrightarrow \log_2 8 = 3$ $\qquad 5^2 = 25 \leftrightarrow \log_5 25 = 2$ $\qquad 10^5 = 10\,000 \leftrightarrow \log_{10} 10000 = 5$

$\log_3 9 = 2 \leftrightarrow 3^2 = 9$ $\qquad \log_4 4096 = 6 \leftrightarrow 4^6 = 4096$ $\qquad \log_{10} 10 = 1 \leftrightarrow 10^1 = 10$

Example #1: Evaluate without using a calculator.

common log
see bottom of page.

a) $\log_8 64$

Let $y = \log_8 64$
then $8^y = 64$
$8^y = 8^2$
So $y = 2$

b) $\log_2 \frac{1}{64}$

Let $x = \log_2 \frac{1}{64}$
then $2^x = \frac{1}{64}$
$2^x = 2^{-6}$
So $x = -6$

c) $\log 100$

Let $m = \log 100$
then $10^m = 100$
$10^m = 10^2$
$m = 2$

d) $\log 10^{-2.6}$

Let $n = 10^{-2.6}$
then $10^n = 10^{-2.6}$
So $n = -2.6$

Example #2: Approximate each of the following:

a) $\log_2 5$ (using systematic trial)

Let $y = \log_2 5$
So $2^y = 5$
check: $2^2 = 4$
$2^3 = 8$
$2^{2.5} = 5.65$
$2^{2.3} = 4.92$
$2^{2.35} = 5.098$
$2^{2.33} = 5.028$
$2^{2.32} = 4.993$
So $\log_2 5 \doteq 2.32$

b) $\log_5 15$ (using intersection)

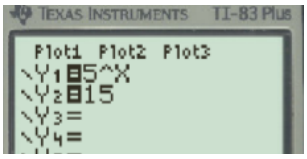
Let $x = \log_5 15$
So $5^x = 15$
Graph: $Y1 = 5^x$, $Y2 = 15$ } see below
FIND POI:
$x = 1.68$

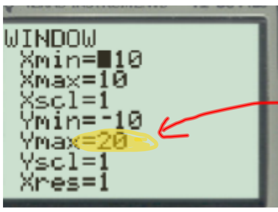
c) $\log_{10} 81$

$= \log 81$
$= 1.908$
(use log button on calc)
only works for base 10

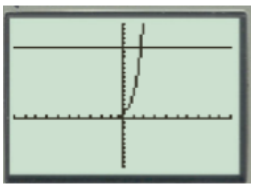
Logarithms to the base 10 are called common logarithms. When writing a common logarithm, it is not necessary to write the base; **log 100 means $\log_{10} 100$**.

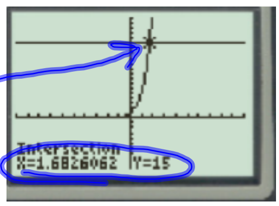
Homework: pg 328; #(1 – 4)odd, 5, 6, 7, 8odd, 10, 13, 14,





need max value greater than 15





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