7.2 Techniques for Solving Exponential Equations

Example #1: Solve

a) $7 = 2(1.06)^{2t+1}$

$\frac{7}{2} = 1.06^{2t+1}$

$\log\left(\frac{7}{2}\right) = \log(1.06)^{2t+1}$

$\log\frac{7}{2} = (2t+1)\log 1.06$

$\frac{\log\frac{7}{2}}{\log 1.06} = 2t+1$

$\frac{\log\frac{7}{2}}{\log 1.06} - 1 = 2t$

$\frac{\left(\frac{\log\left(\frac{7}{2}\right)}{\log(1.06)} - 1\right)}{2} = t$

$10.25 \doteq t$

b) $7^{2x-3} = 2^{-3x}$

Check:

$LS = 7$

$RS = 2(1.06)^{2(10.25)+1}$

$\doteq 7.000129\ldots$

So $LS \doteq RS$.

$\therefore t \doteq 10.25$.

c) $3^{2x} - 4(3)^x + 1 = 0$ (use Quadratic Formula)

Let $m = 3^x$

$m^2 - 4m + 1 = 0$

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-4) \pm \sqrt{(-4)^2-4(1)(1)}}{2(1)}$

$= \frac{4 \pm \sqrt{12}}{2}$

So:

$3^x = \frac{4+\sqrt{12}}{2}$ or $3^x = \frac{4-\sqrt{12}}{2}$

$x\log 3 = \log\frac{4+\sqrt{12}}{2}$ $\quad x\log 3 = \log\left(\frac{4-\sqrt{12}}{2}\right)$

$x = \frac{\log\left(\frac{4+\sqrt{12}}{2}\right)}{\log 3}$ $\quad x = \frac{\log\left(\frac{4-\sqrt{12}}{2}\right)}{\log 3}$

$\doteq 1.1987$ $\quad \doteq -1.1987$

Key Concepts:

- An equation maintains balance when the common logarithm is applied to both sides
- The power law of logarithms ($\log_b x^n = n\log_b x$, $b>0$, $x>0$) is useful tool for solving a variable that appears as part of an exponent.

Steps:

1. Isolate the term containing the variable on one side of the equation.
2. Take the base 10 logarithms of each side of the equation.
3. Apply the power law of logarithms to rewrite the equation without exponents.
4. Solve for the variable and check results.

Homework: pg 375; # 1 – 15

Five Methods to Solve the Exponential Equation $2^{x+1} = 8^{2x}$

Solution 1: Using Base 10 logarithms

Solution 2: Using Base 2 logarthims

Solution 3: By using Exponent Rules

Solution 4: Using TI-83+ and POI

Solution 5: Using TI-83+ and Zero Function

b) $7^{2x-3} = 2^{-3x}$

$\log 7^{2x-3} = \log 2^{-3x}$

$(2x-3)\log 7 = -3x \log 2$

$2x\log 7 - 3\log 7 = -3x\log 2$

$2x\log 7 + 3x\log 2 = 3\log 7$

$x(2\log 7 + 3\log 2) = 3\log 7$

$x = \dfrac{3\log 7}{2\log 7 + 3\log 2}$

$\doteq 0.98$

$2x + 3x$

$= (2+3)x$

$\Rightarrow \dfrac{\log 7^3}{\log 7^2 + \log 2^3}$

$= \dfrac{\log 243}{\log 49 + \log 8}$

Attachments

solution1.pps

solution2.pps

solution3.pps

solution4.pps

solution5.pps