7.4 Techniques for Solving Logarithmic Equations

- It is possible to solve an equation involving logarithms by expressing both sides of a logarithm of the same base: if a = b, then log a = log b, and if log a = log b, then a = b.
- When a quadratic equation is obtained, methods such as factoring or applying the quadratic formula may be used.
- Some algebraic methods of solving logarithmic equations lead to extraneous roots that are not valid solutions to original equations.

Example #1: Find the roots of each equation:

a) $\log(5 - x) = 3$

$10^3 = 5 - x$

$1000 = 5 - x$

$x = -995$

$\log(5-x) = 3$

$\log(5-x) = \log 10^3$

$5 - x = 10^3$

$\vdots$

b) $\log_4(2x + 6) = 3$

$4^3 = 2x + 6$

$64 = 2x + 6$

$58 = 2x$

$29 = x$

Example #2: Solve. Identify and reject any extraneous roots.

a) $\log(x + 2) - 1 = -\log(x - 1)$

~~$x > -2$~~

$x > 1$

$\log(x+2) + \log(x-1) = 1$

$\log(x+2)(x-1) = \log 10$

$(x+2)(x-1) = 10$

$x^2 + x - 2 - 10 = 0$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

So $x = -4$, or $x = 3$

cannot have this as an answer since $x > 1$, so $x = 3$

b) $\log\sqrt[5]{x^2 - 21x} = \frac{2}{5}$

$10^{\frac{2}{5}} = (10^2)^{\frac{1}{5}}$

$\log(x^2 - 21x)^{\frac{1}{5}} = \log 10^{\frac{2}{5}}$

$(x^2 - 21x)^{\frac{1}{5}} = 10^{\frac{2}{5}}$

$x^2 - 21x = 10^2$

$x^2 - 21x - 100 = 0$

$(x-25)(x+4) = 0$

$x = 25$ or $x = -4$

Restrictions

$x^2 - 21x > 0$

$x(x - 21) > 0$

$x < 0$ or $x > 21$

So both answers are possible

$\therefore x = 25$ & $x = -4$

c) $\log_9(x - 5) + \log_9(x + 3) = 1$

(solve on back of page)

Homework: pg 391; #(1 – 3)odd, 5 – 7, 9 – 11

c) $\log_9(x-5)+\log_9(x+3)=1$

$\log_9(x-5)(x+3)=\log_9 9$

$(x-5)(x+3)=9$

$x^2-2x-15-9=0$

$x^2-2x-24=0$

$(x-6)(x+4)=0$

So $x=6$ or $x=-4$

Restrictions: $x>5$

~~$x>-3$~~

So $x=-4$ is not possible

$\therefore x=6$.