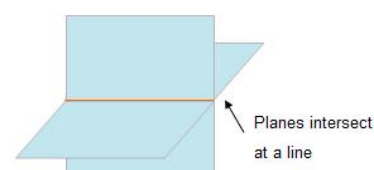


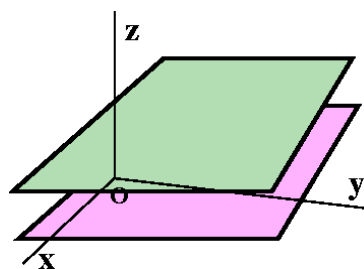
## L8(9.3) Intersections of Two Planes

1- If two planes intersect along a line,  
***the system has an infinite number of solutions described by the parametric equations of the line.***



2- If two planes are co-incident,  
***the system has an infinite number of solutions described by one of the two given equations of the plane.***

3- If two planes are parallel and distinct,  
**the system has no solution.**



Example 1: Solve each system and give a geometric description of the planes.

$$\begin{aligned} \text{a) } & x + 4y - 3z + 6 = 0 & \vec{n}_1 &= (1, 4, -3) \\ & 2x + 8y - 6z + 11 = 0 & \vec{n}_2 &= (2, 8, -6) \end{aligned}$$

$$2\vec{n}_1 = \vec{n}_2$$

At THIS TIME EITHER Parallel  OR COINCIDENT 

So check D values:  $\because 2(6) \neq 11 \quad \therefore$  The planes are parallel & distinct.

$\therefore$  No solutions.

$$\begin{aligned} \text{b) } \pi_1: & 5x - y + 2z - 9 = 0 \\ \pi_2: & -25x + 5y - 10z + 45 = 0 \end{aligned}$$

Since  $-5\pi_1 = \pi_2 \Rightarrow$  Planes are coincident

$\therefore$  Infinite solutions

$$\begin{array}{r} \text{c) } 4x + 7y - 33z + 17 = 0 \quad \textcircled{1} \times 2 \\ 8x + 5y - 3z + 7 = 0 \quad \textcircled{2} \\ \hline \end{array}$$

$$\textcircled{3} \quad 9y - 63z + 27 = 0$$

~~→ The equation of line of intersection.~~

oops ï

Note: I made a mistake with this example - See example 3 on p.512 - You need to substitute a parameter in for z and solve the parametric equations for x and y (Observe below)

At this point, to determine the equation of the line of intersection, a parameter must be introduced. From equation 3 we let  $z = s$

$$\begin{aligned} \rightarrow 9y - 63(s) + 27 &= 0 \\ y &= 7s - 3 \end{aligned}$$

At this point substitute  $z = s$  and  $y = 7s - 3$  into equation 1

$$\begin{aligned} 4x + 7y - 33z + 17 &= 0 \\ 4x + 7(7s - 3) - 33(s) + 17 &= 0 \\ 4x + 49s - 21 - 33s + 17 &= 0 \end{aligned}$$

$$x = -4s + 1 \quad \text{and isolate for } x$$

Therefore the parametric equations of the line of intersection are

$$x = -4s + 1, \quad y = 7s - 3, \quad \text{and } z = s, \quad s \in \mathbb{R}$$

I apologize.

# Assigned Work

p.516 #6, 8