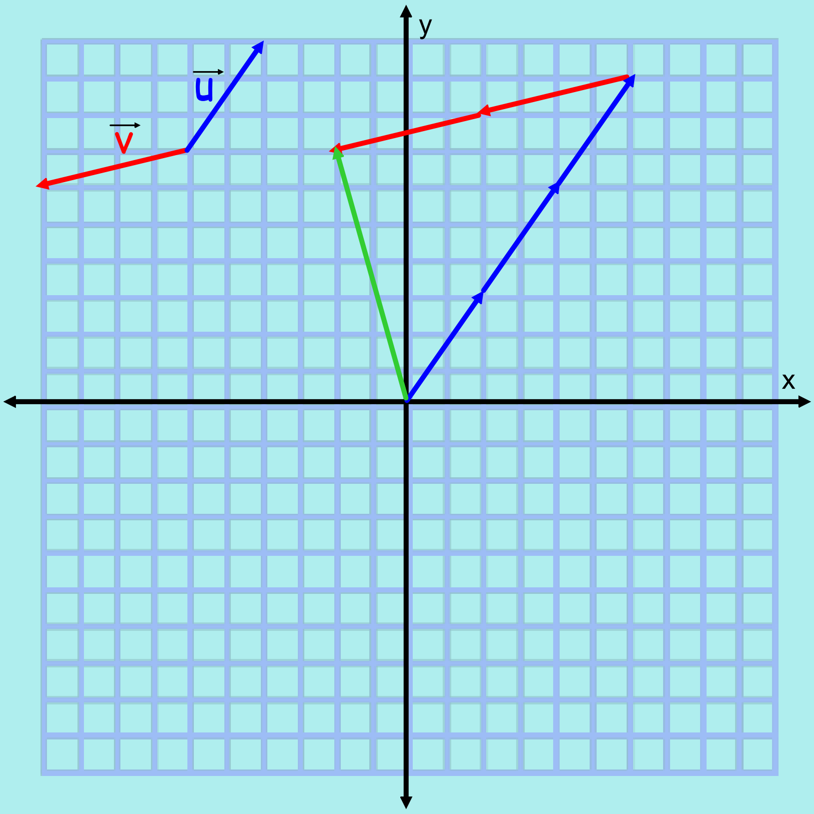
L8(6.8) Linear Combinations & Spanning Sets

Given the vectors $\vec{u}$ and $\vec{v}$, draw $3\vec{u} + 2\vec{v}$.



The resultant $3\vec{u} + 2\vec{v}$ is said to be a linear combination of the vectors $\vec{u}$ and $\vec{v}$.

Ex1: Show that the vector $\vec{w} = (4, 23)$ can written as a linear combination of the vectors $\vec{u} = (-1, 4)$ and $\vec{v} = (2, 5)$.

$\vec{w}$

If $\vec{w}$ is a linear combination of vectors $\vec{u}$ and $\vec{v}$, then let a & b be scalar multiples of $\vec{u}$ & $\vec{v}$ respectively

$$a\vec{u} + b\vec{v} = \vec{w}$$

$$a(-1,4) + b(2,5) = (4,23)$$

① $-a + 2b = 4$

① $\times 4$ $\quad -4a + 8b = 16$

② $4a + 5b = 23$

① $\times 4 +$ ② $\quad 13b = 39$

$b = 3$

From ① $\quad a = 2$

So, $2\vec{u} + 3\vec{v} = \vec{w}$

In Summary for $\mathbf{R}^2$

Any pair of nonzero, noncollinear vectors will span $\mathbf{R}^2$.

Any vector, $\overrightarrow{OP}$, in $\mathbf{R}^2$, can be written uniquely as a linear combination of the standard basis vectors (ie: $\vec{i}$ and $\vec{j}$ span $\mathbf{R}^2$):

$$\overrightarrow{OP} = (a, b) = a(1, 0) + b(0, 1)$$

In Summary for $\mathbf{R}^3$

Any pair of nonzero, noncollinear vectors will span a plane in $\mathbf{R}^3$. This means that any vector in that plane can be expressed as a linear combination of these two vectors.

Any vector, $\overrightarrow{OP}$, in $\mathbf{R}^3$, can be written uniquely as a linear combinationof the three standard $\vec{\text{basis}}$ $\text{vec}\vec{\text{to}}\text{rs}$ (ie: i, j, and k span $\mathbf{R}^3$):

$$\overrightarrow{OP} = (a, b, c) = a(1, 0, 0) + b(0, 1, 0) + c(0, 0, 1)$$

Ex2: Show that the vector $\vec{w}$ (-9, -4, 1) lies in a plane determined by the vectors $\vec{u}$ = (-1, -2, 1) and $\vec{v}$ = (3, -1, 1).

Let a & b represent scalars

We need to show that

$$a\vec{u} + b\vec{v} = \vec{w}$$

$$a(-1,-2,1) + b(3,-1,1) = (-9,-4,1)$$

① $-a + 3b = -9$

② $-2a - b = -4$

③ $a + b = 1$

Solution is possibly $3\vec{u} - 2\vec{v} = \vec{w}$

①+③ ⇒ $4b = -8$

$b = -2$

From ③ $a = 3$

Check w equation ②

LS	RS
$-2a-b$	-4
$= -2(3)-(-2)$	
$= -4$	

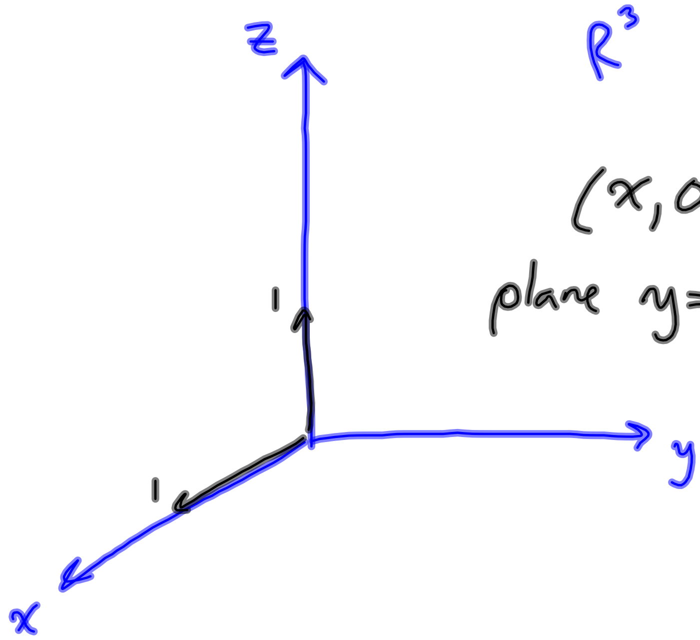
LS=RS

∴ $\vec{w}$ can be written as a linear combination $\vec{w} = 3\vec{u} - 2\vec{v}$,

∴ The vector (-9,-4,1) lies in the plane determined by $\vec{u}$ & $\vec{v}$.

Ex3: The vector set {(1, 0, 0), (0, 0, 1)} spans a set in $\mathbf{R}^3$.
Describe this set.

R^3



$(x, 0, z)$

plane $y=0$

Assigned Work:

p.340 #6, 7b, 8, 10, 11, 13, 14

& Read all examples
p. 335 to 339.