Homework Take-up

L8 (2.3) The Product Rule

$$\boxed{\text{If } p(x) = f(x)g(x), \text{ then } p'(x) = f'(x)g(x) + g'(x)f(x)}$$

Proof of Product Rule:

(also on p.85)

http://math.ucsd.edu/~wgarner/math20a/prodrule.htm

Given that p(x) = f(x)g(x) we can use the limit definition of the derivative to write

$$p'(x) = \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x+h) + f(x)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x+h)}{h} + \lim_{h\to 0} \frac{f(x)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \underbrace{\left\{\frac{f(x+h) - f(x)}{h}\right\}}\, g(x+h) + \lim_{h\to 0} \left\{\frac{g(x+h) - g(x)}{h}\right\} f(x)$$

$$\boxed{p'(x) = f'(x)\,g(x) + g'(x)\,f(x)}$$

Ex1: Find the derivative of each function. & simplify

a) $y = \underbrace{(2x+4)}_{f(x)}\underbrace{(3x-5)}_{g(x)}$

$y' = f'(x)\,g(x) + f(x)\,g'(x)$

$y' = 2(3x-5) + 3(2x+4)$

$= 6x - 10 + 6x + 12$

$= 12x + 2$

b) $f(x) = (3x^2 + 4x - 6)(2x^2 - 3x - 9)$

$f'(x) = (6x+4)(2x^2-3x-9) + (4x-3)(3x^2+4x-6)$

$$\begin{aligned}
= {} & 12x^3 - 18x^2 - 54x \\
& 12x^3 + 8x^2 - 12x - 36 \\
& +16x^2 - 24x \\
& -9x^2 - 12x + 18 \\
\hline
= {} & 24x^3 - 3x^2 - 102x - 18
\end{aligned}$$

Ex2: Find the point(s) on the curve $f(x) = \overbrace{2(x - 1)}^{h(x)}\overbrace{(5 - x)}^{g(x)}$ where the tangent is horizontal. What is the equation of the tangent line?

$$f'(x) = 2(1)(5-x) + 2(x-1)(-1)$$

$$= 10-2x -2x+2$$

$$f'(x) = -4x+12$$

set $f'(x)=0$

$$0 = -4x+12$$

$$-12 = -4x$$

$$x = 3$$

To find point:
sub $x=3$ into $f(x)$

$$f(3) = 2(3-1)(5-3)$$

$$= 8$$

The point is $(3,8)$.

$$y = \cancel{m}^{0}x+b$$

$$y = b$$

$$8 = b$$

$\therefore$ The equation of the tangent line is $y=8$.

Assigned Work:

p. 90-91 #1abcde, 5abc, 6, 7, 9