

1. Use the power rule to find the derivative

of $y = 3x^5 - \frac{3}{x^2} + \sqrt{x}$

$$y = 3x^5 - 3x^{-2} + x^{1/2}$$

$$y' = 15x^4 + 6x^{-3} + \frac{1}{2}x^{-1/2}$$

$$y' = 15x^4 + \frac{6}{x^3} + \frac{1}{2\sqrt{x}}$$

2. Use the product rule to find the derivative

of $y = (2x^2 + 5)(3x - 10)$

$$y' = 4x(3x - 10) + 3(2x^2 + 5)$$

$$= 12x^2 - 40x + 6x^2 + 15$$

$$= 18x^2 - 40x + 15$$

$$5c) y = (3 - 2x - x^2)(x^2 + x - 2)$$

$$p.90 \quad \frac{dy}{dx} = (-2 - 2x)(x^2 + x - 2) + (2x + 1)(3 - 2x - x^2)$$

$$= -2x^2 - 2x + 4 - 2x^3 - 2x^2 + 4x + 6x - 4x^2 - 2x^3 + 3 - 2x - x^2$$

$$= -4x^3 - 9x^2 + 6x + 7$$

$$\frac{dy}{dx} \Big|_{x=2} = -4(-2)^3 - 9(-2)^2 + 6(-2) + 7$$

$$= 32 - 36 - 12 + 7$$

$$= -9$$

9.
p.71

$$V(t) = 75 \left(1 - \frac{t}{24}\right)^2, \quad 0 \leq t \leq 24$$
$$= 75 \left(1 - \frac{t}{24}\right) \left(1 - \frac{t}{24}\right) \quad V = 0.60 \times 75 \text{ L}$$

$$V'(t) = 75 \left(-\frac{1}{24}\right) \left(1 - \frac{t}{24}\right) + 75 \left(-\frac{1}{24}\right) \left(1 - \frac{t}{24}\right)$$
$$= 150 \left(-\frac{1}{24}\right) \left(1 - \frac{t}{24}\right)$$

To Find t : sub $V = 0.6 \times 75$ into $V(t)$:

$$0.6 \times 75 = 75 \left(1 - \frac{t}{24}\right)^2$$

$$0.6 = \left(1 - \frac{t}{24}\right)^2$$

$$\pm \sqrt{\frac{6}{10}} = 1 - \frac{t}{24}$$

$$\frac{t}{24} = 1 \pm \sqrt{\frac{6}{10}}$$

$$t = 24 \left(1 \pm \sqrt{\frac{6}{10}}\right)$$

$$t = 24 \left(1 + \sqrt{\frac{6}{10}}\right) \quad \text{or} \quad t = 24 \left(1 - \sqrt{\frac{6}{10}}\right)$$

$$= 24 + 24\sqrt{\frac{6}{10}} \quad = 24 - 24\sqrt{\frac{6}{10}}$$

$$\doteq 42.6 \quad \sim \text{inadmissible} \quad \doteq 5.4 \quad \sim \text{Kept in mem.}$$

since outside domain.

$$V'(5.4) \doteq 150 \left(-\frac{1}{24}\right) \left(1 - \frac{5.4}{24}\right)$$

$$\doteq -4.84$$


\therefore The gas is leaking out of the tank after approx 5.4 hours at a rate of $4.84 \frac{\text{L}}{\text{h}}$.

L9 (2.4) The Quotient Rule

$$\text{If } q(x) = \frac{f(x)}{g(x)}, \text{ then } q'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2},$$

where $g(x) \neq 0$.

NOTE: Proof of Quotient Rule can be found on p.94

$$q(x) = \frac{f(x)}{g(x)}$$


$$g(x)q(x) = f(x)$$

Ex1: Find the derivative of each function.

$$\text{a) } y = \frac{3x}{x^2 + 5}$$

$f(x)$
 $g(x)$

$$\begin{aligned} y' &= \frac{3(x^2 + 5) - 2x(3x)}{(x^2 + 5)^2} \\ &= \frac{3x^2 + 15 - 6x^2}{(x^2 + 5)^2} \\ &= \frac{-3x^2 + 15}{(x^2 + 5)^2} \end{aligned}$$

$$\text{b) } y = \frac{x^2 - 4x - 12}{x^3 - 8}$$

$$\begin{aligned} y' &= \frac{(2x - 4)(x^3 - 8) - (3x^2)(x^2 - 4x - 12)}{(x^3 - 8)^2} \\ &= \frac{2x^4 - 16x - 4x^3 + 32 - 3x^4 + 12x^3 + 36x^2}{(x^3 - 8)^2} \\ &= \frac{-x^4 + 8x^3 + 36x^2 - 16x + 32}{(x^3 - 8)^2} \end{aligned}$$

NOTE: The Quotient Rule is not needed when the denominator of a function is a monomial.

$$\text{ex: } y = \frac{x^2 - 5x}{3} \quad \text{or} \quad y = \frac{x^3 + 1}{x^2}$$

Ex2: Find the coordinate(s) on the curve $f(x) = \frac{2x+8}{\sqrt{x}}$
where the tangent line is horizontal.

$$f'(x) = \frac{2(\sqrt{x}) - \frac{1}{2}x^{-1/2}(2x+8)}{(\sqrt{x})^2}$$

$$= \frac{2\sqrt{x}}{x} - \frac{2x+8}{2x\sqrt{x}}$$

$$= \frac{4x - 2x - 8}{2x\sqrt{x}}$$

$$= \frac{2x - 8}{2x\sqrt{x}}$$

$$= \frac{x-4}{x\sqrt{x}}$$

Set $f'(x) = 0$ to solve for x

$$0 = \frac{x-4}{x\sqrt{x}}$$

$$0 = x-4$$

$$x=4$$

$$f(4) \dots$$

Assigned Work:

p. 97 #4, 5cd, 6, 7, 8, 9, 13