p. 532

#13b)

$$\textcircled{1}\ 2x - y + z = -3$$
$$\textcircled{2}\ x + y - 2z = 1$$
$$\textcircled{3}\ 5x + 2y - 5z = 0$$

None of the planes are parallel

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 2 & -1 & 1 & -3 \\ 5 & 2 & -5 & 0 \end{array}\right] \sim \begin{array}{c} \\ 2R_1 - R_2 \\ 5R_1 - R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 3 & -5 & 5 \\ 0 & 3 & -5 & 5 \end{array}\right]$$

$$\sim \begin{array}{c} \\ R_2 \div 3 \\ R_2 - R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 1 & -\frac{5}{3} & \frac{5}{3} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Three planes intersect in one line: <u>consistent</u>

$0z = 0 \quad z = t \quad t \in \mathbb{R}$ then Back Solve:

$$y - \frac{5}{3}z = \frac{5}{3}$$
$$y = \frac{5}{3} + \frac{5}{3}t$$

$$x + y - 2z = 1$$
$$x + \left(\frac{5}{3} + \frac{5}{3}t\right) - 2t = 1$$
$$x = -\frac{2}{3} + \frac{1}{3}t$$

$\therefore$

$\vec{r} = \left(-\frac{2}{3}, \frac{5}{3}, 0\right) + t\left(\frac{1}{3}, \frac{5}{3}, 1\right), t \in \mathbb{R}.$

is the line of intersection.

CALCULUS REVIEW

TAKING DERIVATIVES:

① Power rule: $y = x^n \rightarrow y' = n x^{n-1}$

② Prod rule: $y = f \cdot g \rightarrow y' = f'g + g'f$

③ Quotient rule: $y = \frac{f}{g} \rightarrow y' = \frac{f'g - g'f}{g^2}$

④ Chain rule: $y = (f)^n \rightarrow y' = n(f)^{n-1} f'$

⑤ Euler exponential: $y = e^{g(x)} \rightarrow y' = e^{g(x)} g'(x)$

⑥ Natural Logarithm: $y = \ln(g(x)) \rightarrow y' = \frac{1}{g(x)} \cdot g'(x) = \frac{g'(x)}{g(x)}$

⑦ GENERAL EXPONENTIALS: $y = b^{(g(x))} \rightarrow y' = b^{g(x)} \cdot \ln b \cdot g'(x)$

⑧ GENERAL LOGARITHMS: $y = \log_b(g(x)) \rightarrow y' = \frac{1}{g(x)} \cdot \frac{1}{\ln b} \cdot g'(x) = \frac{g'(x)}{g(x) \ln b}$

DERIVS OF TRIG FUNCTIONS

⑨ Sinusoidal: $y = \sin(g(x)) \rightarrow y' = \cos(g(x)) g'(x)$

⑩ Cosine: $y = \cos(g(x)) \rightarrow y' = -\sin(g(x)) g'(x)$

⑪ Tangent: $y = \tan(g(x)) \rightarrow y' = \sec^2(g(x)) g'(x)$

Try to combine rules in one function

" Take the deriv. in one line → DO NOT SIMPLIFY"

$$y = \left[\frac{\cos(3x^2)}{\log_2(\sin(e^{3x}))}\right]^3$$

$$y' = 3\left[\quad " \quad\right]^2 \cdot$$

$$\frac{-\sin(3x^2)(6x)\log_2(\sin(e^{3x})) - \frac{3\cos(e^{3x})e^{3x}\cos(3x^2)}{\sin(e^{3x})\cdot\ln 2}}{\left(\log_2(\sin(e^{3x}))\right)^2}$$

OPTIMIZATION

WORD PROBS

$V = 2\pi r^2 h \quad \left(h = \frac{1}{3r^3}\right)$

$V = 2\pi r^2 \left(\frac{1}{3r^3}\right)$

$V(r) = \frac{2\pi}{3r}$

$V'(r) = \frac{-2\pi}{3r^2}$

set $V'(r) = 0$
Also check endpoints

CURVE SKETCHING

$y = V$

$y' = V'$

what are you finding when you set $y' = 0$

CRIT. VALUES (x's only)

Sub to get crit points if asked $(x, f(x))$

FOR SUMMATIVE STUDY THE FOLLOWING:

- All tests
- All derivs
- Curve Sketching ← Algorithm not given (see p. 207)

↳ This includes asymp using limits.

Optimization as it relates to Curve Sketching.

$$y=\frac{x}{x^2-4x+4}=\frac{x}{(x-2)^2}=\frac{x}{(x-2)(x-2)}$$

① $D=\{x\in\mathbb{R},\ x\neq 2\}$

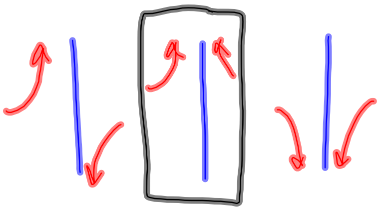
② Intercepts

$f(0)=0 \quad \therefore (0,0)$

③ Asymptotes:

V.A. @ $x=2$

behaviours



$\lim\limits_{x\to 2^-} f(x) \cong \infty$ ← use 1.9999

$\lim\limits_{x\to 2^+} f(x) \cong \infty$ ← 2.00001

H.A. @ $y=0$ MHF4U

END Behaviours

$\lim\limits_{x\to \infty} f(x) = 0$ ↑result +ve → $y=0$

$\lim\limits_{x\to -\infty} f(x) = 0$ ↑result −ve

} $y=0$

④ Find Crit Points:

$$y=\frac{x}{(x-2)^2}$$

$$y'=\frac{1(x-2)^2-2(x-2)(1)(x)}{(x-2)^4}$$

$$=\frac{\cancel{(x-2)}[x-2-2x]}{(x-2)^{\cancel{4}\,3}}$$

$$=\frac{-x-2}{(x-2)^3}$$

Set $y'=0$ to find crit values:

$x=-2$ is the crit value

get point:

$$f(-2)=\frac{-2}{(-2-2)^2}=-\frac{1}{8}$$

$\therefore \left(-2,-\frac{1}{8}\right)$ is crit point

[2nd Deriv Test]

$$F''\left(-\tfrac{1}{2}\right)=\begin{cases}+ve & \text{MIN}\\ -ve & \text{MAX}\end{cases}$$

⑤ Table of Inc/Dec $\bar{c}$ $f'(x)$

	$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
	$x=-2$		$x=2$
$-x-2$	+	−	−
$(x-2)^3$	−	−	+
$f'(x)$	−	+	−
$f(x)$	Dec	Inc	Dec

MIN @ $\left(-2,-\frac{1}{8}\right)$ V.A. NOT A MAX !!

⑥ Inflection Pts:

$$y' = \frac{-x-2}{(x-2)^3}$$

$$y'' = \frac{-(x-2)^3 - 3(x-2)^2(1)(-x-2)}{(x-2)^6}$$

$$= \frac{\cancel{(x-2)^2}\left[-x+2+3x+6\right]}{(x-2)^{\cancel{6}\,4}}$$

$$= \frac{2x+8}{(x-2)^4}$$

set $y''=0$ $x=-4$

$f(-4) = -\frac{1}{9}$

$\therefore$ Inf pt is $\left(-4,-\frac{1}{9}\right)$.

⑦ Table of Concavity: $\bar{c}$ $f''(x)$

	$(-\infty,-4)$	$(-4,2)$	$(2,\infty)$
	$x=-4$		$x=2$
$2x+8$	−	+	+
$(x-2)^4$	+	+	+
$f''(x)$	−	+	+
$f(x)$	Down	UP	UP

⑧ Sketch w/ all info