Key Concepts to Review:

- Position, Velocity & Acceleration
- Perimeter, Area & Volume Optimization
- Distance Optimization
- Time Optimization
- Economics: Profit, Revenue, Cost Optimization

Important Points to remember

Position, Velocity & Acceleration

- Usually given function
- Use units to help build function otherwise

© Be sure to communicate proper units. m, km, $\frac{m}{s}$, km/hr, m/s^2 km/hr^2

© $s(t)$, $v(t)$, $a(t)$

$s(t) = 3t^3 + 2t - 5$

$t \in \mathbb{R}, 0 \leq t \leq 7$

$[0, 7]$

- endpoints
- local max, mins
- absolute max, mins
- turning points
- moving towards or away from starting position (s.v).

Perimeter, Area, Volume. → OPTIMIZATION

USE STEPS TO FASCILITATE SOLVING THESE PROBLEMS

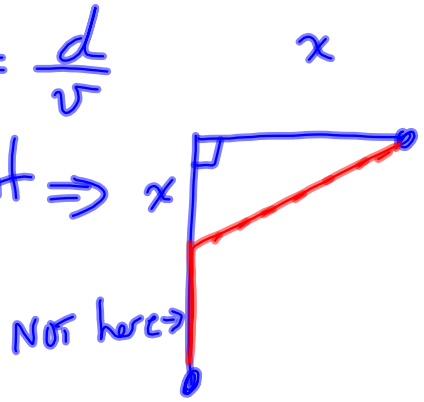
- Draw the diagram of situation/object
- Relate quantities that vary to each other
- State the equation you need to optimize. (Check units)
- Take deriv. and set equal to zero.
- Answer the question (endpoints?).

Distance Optimization: – same as before.

- $d = v \cdot t$
- watch out for inadmissable answers

Time Optimization

- $t = \frac{d}{v}$



Economics

Minimizing Cost $C(x) \Rightarrow$

Maximizing Revenue

MAX $P(x) = R(x) - C(x)$

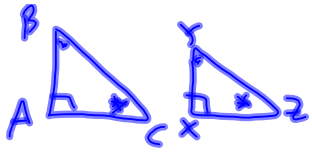
- Watch your capacities / domain
- Watch units.

Review

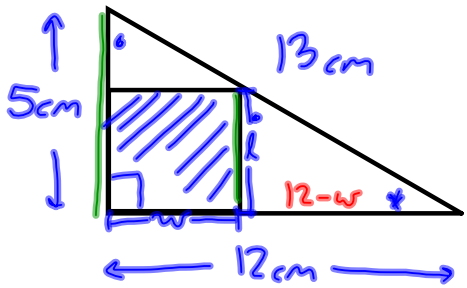
p.156-159
#3, 5, 6, 7ab, 8, 9,14,
15, 17, 19 to 24, 26d

→ This review is missing time optimization question.

p. 146 #12



$\frac{XY}{AB} = \frac{XZ}{AC} = \frac{YZ}{BC}$



$\frac{l}{5} = \frac{12-w}{12}$

$l = \frac{5(12-w)}{12}$

$A = l \cdot w$

$A(w) = \frac{5(12-w)}{12} \cdot w$

$= \left[\frac{60}{12} - \frac{5w}{12}\right] w$

$= 5w - \frac{5w^2}{12}$

$A'(w) = 5 - \frac{10}{12} w$

$0 = 5 - \frac{10}{12} w$

$\frac{10}{12} w = 5$

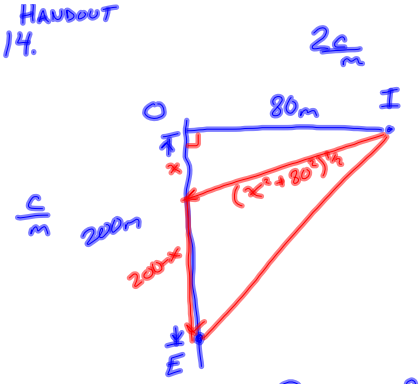
$w = \frac{60}{10}$

$= 6$

$l = \frac{5(12-6)}{12}$

$= 2.5$

#14. HANDOUT



$Cost_{OE} = 80 \cdot 2c + 200c$

$C(x) = 2c(x^2+80^2)^{1/2} + c(200-x)$

$C'(x) = c(x^2+80^2)^{-1/2}(2x) - c$

$0 = \frac{2xc}{(x^2+80^2)^{1/2}} - c$

$c = \frac{2xc}{(x^2+80^2)^{1/2}}$

$1 = \frac{2x}{(x^2+80^2)^{1/2}}$

$(x^2+80^2)^{1/2} = 2x$

$x^2+80^2 = 4x^2$

$3x^2 = 80^2$

$x^2 = \frac{80^2}{3}$

$x \doteq \pm 46.3$ but $x = -46.3$ ∴ extraneous. ∴ inadmissable.

Length of cable in water $= (x^2+80^2)^{1/2}$

$\doteq (46.3^2+80^2)^{1/2}$

$\doteq 92.4$

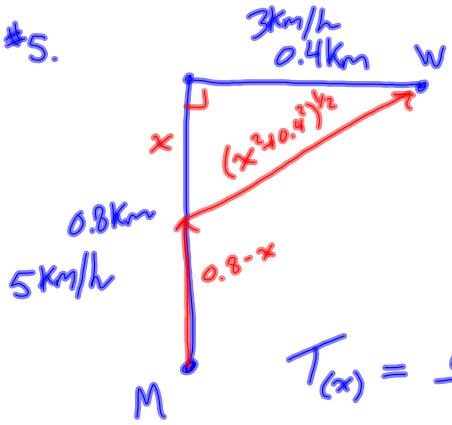
Length on land $= 200-x$

$\doteq 200-46.3$

$\doteq 153.7$

∴ The cable should be laid ∎ 92.4 m under water and 153.7m on land.

#5.



OPTIMIZING TIME PROBLEM

$t = \frac{d}{v}$

$$T(x) = \frac{0.8-x}{5} + \frac{(x^2+0.4^2)^{1/2}}{3}$$

$$= \frac{0.8}{5} - \frac{x}{5} + \frac{(x^2+0.4^2)^{1/2}}{3}$$

$$T'(x) = -\frac{1}{5} + \frac{1}{3}\cdot\frac{1}{2}(x^2+0.4^2)^{-1/2}(2x)$$

Set $T'(x) = 0$

$$\frac{1}{5} = \frac{x}{3(x^2+0.4^2)^{1/2}}$$

$$3(x^2+0.4^2)^{1/2} = 5x$$

$$9(x^2+0.4^2) = 25x^2$$

$$25x^2 - 9x^2 - 9(0.4^2) = 0$$

$$16x^2 = 9(0.4)^2$$

$$x^2 = \frac{9(0.16)}{16}$$

$$x^2 = 0.09$$

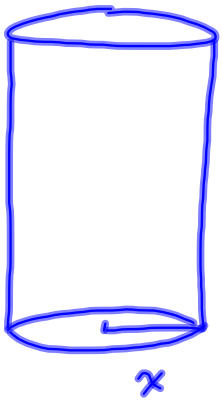
$$x = \pm 0.3$$

$x = 0.3$ $x = -0.3$ is erroneous.

$T(0.3) \doteq 0.27$

∴ The fastest will take to get to the wreck is approx. 0.27 hours.

3.



$S.A. = 270\pi \text{ cm}^2$

$270 = 2\pi r^2 + 2\pi rh$

$h = \frac{270 - 2\pi r^2}{2\pi r}$

$V = \pi r^2 \cdot h$

$V(r) = \pi r^2 \cdot \left[\frac{270 - 2\pi r^2}{2\pi r}\right]$

$= 135r - \pi r^3$

$V'(r) = 135 - 3\pi r^2$

Set $V'(r) = 0$

$0 = 135 - 3\pi r^2$

$r = \pm\sqrt{\frac{135}{3\pi}}$

$\doteq \ldots$

p.641
#23

25 Good