Grade 11 Mathematics   Functions (MCR3U)

Financial Literacy in Grade 11 Mathematics   Understanding Annuities

Connections to Financial Literacy

Students are building their understanding of financial literacy by solving problems related to annuities. Students set up a hypothetical regular payment annuity for savings and identify life circumstances that might impact such an annuity. Students determine monthly payments and interest associated with a mortgage.

Curriculum Expectations	Learning Goals
Click here to access expectations in full, with examples. **C. Discrete Functions** **3. Solving Problems Involving Financial Applications** **C3.** make connections between sequences, series, and financial applications, and solve problems involving compound interest and ordinary annuities **3.7** solve problems, using technology, that involve the amount, the present value, and the regular payment of an ordinary simple annuity. **Mathematical Process Expectations** • Problem Solving • Selecting Tools and Computational Strategies • Connecting • Communicating	**During this lesson, students will:** • solve problems involving regular payments in simple annuities • use scientific or graphing calculators to solve problems using the $$A = \frac{R\left[\left(1+i\right)^{n} - 1\right]}{i} \quad \text{and}$$ $$PV = \frac{R\left[1 - \left(1+i\right)^{-n}\right]}{i} \quad \text{formulas}$$ • communicate solutions to annuity problems in writing • make connections between the mathematical equations and real life situations **Sample Success Criteria** for making connections between mathematical equations and real life situations: Explain how different interest rates and payment schedules have an impact on annuities

Instructional Components and Context

Readiness	Terminology	Materials and Resources
• Understanding the relationship between ordinary simple annuities and geometric series. • Skill in solving the polynomial and exponential equations that are generated when different values are known and unknown in the $$A = \frac{R\left[\left(1+i\right)^{n} - 1\right]}{i} \quad \text{formula.}$$ Skill in using a scientific or graphing calculator to solve linear equations.	• Present value • Amount • Future value • Regular payment • Compounded interest • Compounding rate per period • Annuity	• Textbook or materials with practice questions • Computer and projector • Scientific or graphing calculators • Class Set of Exit Slip

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Minds On	Connections

Whole Class ➔ Check for Understanding

Pose the scenario to the class:

> Suppose a new classmate joined us today. And, suppose that classmate had learned about sequences and series, but had not yet seen applications to financial literacy. How would you explain the formula
>
> $$A = \frac{R\left[\left(1+i\right)^{n} - 1\right]}{i}$$ that we have been using to that student?

Individual ➔ Check for Computational Skills

Students complete the following and check their answer with a partner:

> Calculate the future value and interest earned for an annuity with regular deposits of $500 every 6 months, for 10 years, at a 6.2%/annum interest rate compounded semi-annually.

Whole Class ➔ Preparing for Problem Solving

Pose the following questions:

1. Instead of starting now to put regular payments into an annuity in order to have a certain amount in the future (at the end of the annuity), we need to pay off a certain expense now with the promise of making regular payments into the future. How could we model this scenario using a geometric sequence?
2. Before we create a model, what are some life scenarios where people need to pay off certain expenses now with the promise of regular payments into the future?

Possible answers: a mortgage; a car loan, a present commitment to support a child with future post-secondary expenses

Accept whatever names and variables students use as a beginning step, then gradually introduce *PV* for present value, and explain why the variables *R*, *i*, and *n* still apply. Work towards $PV = R(1+i)^{-1} + R(1+i)^{-2} + \ldots R(1+i)^{-n}$, reviewing geometric sequences as you work with what the students put forward.

A for **L** Observe which students are making the connections between geometric sequences and annuities, and which students need further support so that you can guide those students during the Action part of the lesson.

Guiding Questions

- Provide reminders re: using calculators, and always writing a conclusion at the end of a problem.
- Will you get a more accurate answer if you do the calculation using a decimal or a fraction?

DI Observe which students need further support in substituting into the formula and using their calculator for computation.

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Action!	Connections
Whole Class → Formula Development	**Guiding Questions**

Whole Class → Formula Development

Guide development of the

$$PV = \frac{R\left[1 - \left(1 + i\right)^{-n}\right]}{i}$$ formula

by reviewing the process for summing a geometric sequence and applying a formula for a geometric series.

Lead the class in application of the PV formula using the following example:

> Your parents agree to set up an annuity with the bank now to help you pay for living expenses over the next 5 years. The annuity will pay $50 per month and the first payment will be made one month from now. If the annuity earns 7.75%/annum interest rate compounded monthly:
>
> a. How much money should your parents initially put into the annuity?
> b. How much interest will the annuity earn over its term?

Pairs → Share Practice

Students work with a partner to solve the problem:

> Amir borrows $250 000 for a new condominium. If the bank charges 3.9%/a compounded monthly for the next 25 years:
>
> a. How much will each of Amir's monthly payments be?
> b. How much interest will he have paid over the length of the loan?
>
> What does the interest earned tell you about the banking/loan business?

🔴 **DI** Differentiate support based on observation of need. Circulate to see which pairs are making good progress and which need help. Invite pairs to discuss and assess their solutions, and support peers. Have more complex problems available for students who need an additional challenge.

Guiding Questions

- Is this possible for everyone?
- Why might someone not have funds available to make investment choices like the ones we're discussing?
- What factors might cause a person's financial situation to change?

A for L Teacher observation of student responses during review questions

A as L Student reflection on application of the information to their lives

A of L Student presentation of a case study (occurs in future lesson)

🔴 **DI** Students may select from a variety of problems/case studies to demonstrate their learning

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Consolidation	Connections
Pairs ➔ Complete Exit Slip Students complete the Exit Slip. Follow-up and Practice: **DI** Observe student needs and provide different questions for those who: • need practice on concepts • need practice on skills • need extension beyond classwork.	**A** **as** **L** Peer feedback on the completed Exit Slips at the start of next class

Curriculum Expectations

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Overall Expectations	Specific Expectations
Internal Control and Financial Analysis	
Discrete Functions **3.** make connections between sequences, series, and financial applications, and solve problems involving compound interest and ordinary annuities	**Solving Problems Involving Financial Applications** **C3.7** solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary simple annuity (e.g., calculate the total interest paid over the life of a loan, using a spreadsheet, and compare the total interest with the original principal of the loan).
Mathematical Process Expectations	
Problem Solving	– develop, select, apply, compare, and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
Selecting Tools and Computational Strategies	– select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
Connecting	– make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
Communicating	– communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.
Prior Knowledge – Readiness	
Solving Problems Involving Financial Applications	**3.5** explain the meaning of the term annuity, and determine the relationships between ordinary simple annuities (i.e., annuities in which payments are made at the end of each period, and compounding and payment periods are the same), geometric series, and exponential growth, through investigation with technology (e.g., *use a spreadsheet to determine and graph the future value of an ordinary simple annuity for varying numbers of compounding periods; investigate how the contributions of each payment to the future value of an ordinary simple annuity are related to the terms of a geometric series*)

Exit Slip

a) Valerie is planning to buy a car. She is working on her budget before she starts her search to ensure that she will be able to afford the car she chooses. She can afford to make a payment of $650 per month for the next four years. The interest rate on the car loan will be 9.5% compounded monthly.

When Valerie starts looking for her car, what should she use as her price range?

b) Artur wants to buy a new stereo system that is on sale for $2500. He decides to pay for it using a two-year instalment plan with an interest rate of 8% compounded quarterly.

How much will each of his quarterly payments be?

How much interest is Artur paying?