One important type of potential energy is associated with springs and other elastic objects. In a relaxed state, with no force applied to its end, the spring is at rest.

Suppose you pull on the spring with a force $F_{\text{pull}}$, causing the spring to stretch to the right, as shown. When stretched, the spring exerts a force $F_{\text{spring}}$ to the left.
Spring Forces

Likewise, if you push on the spring with a force $F_{\text{push}}$, it compresses to the position shown. When compressed, the spring exerts a force $F_{\text{spring}}$ to the right.

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Spring Forces

According to Newton's third law of motion, the force exerted by the object that is applying the force to the spring is equal and opposite to the force that the spring exerts on that object. In both cases, $F_{\text{spring}}$ is called the restorative force because it tends to restore the spring to its natural length.

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Spring Forces

NOTE!

The amount of force exerted by a spring is proportional to the spring’s displacement ($\Delta x$). This is Hooke's law, named after Robert Hooke who discovered the relationship in 1678.
Hooke's Law

Hooke's Law is given by the equation:

\[ F_x = k \Delta x \]

where:
- \( F_x \) is the force exerted by the spring (N)
- \( k \) is the spring constant (N/m)
- \( \Delta x \) is the displacement of the spring from its rest position (m)

**NOTE!**
The direction of \( F_x \) is opposite to the direction of the displacement.

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**PRACTICE**
1. Use the F-x graph given to answer the following questions.
   (a) What does the slope of the graph represent?

   (a) the spring constant "k"

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**PRACTICE**
1. Use the F-x graph given to answer the following questions.
   (b) As the stiffness of the spring increases, what happens to the graph?

   (b) as stiffness \( k \) increases, slope \( \frac{F_x}{\Delta x} \) increases
Hooke's Law

PRACTICE

2. A typical compound archery bow requires a force of 133 N to hold an arrow at "full draw" (pulled back 71 cm). Assuming that the bow obeys Hooke's law, what is its spring constant?

\[ k = 190 \text{ N/m} \]

October 14, 2012 4U2 - Elastic Potential Energy

3. A spring hangs at rest from a support. If you suspend a 0.46 kg mass from the spring, its deflection is 7.9 cm.

(a) Determine the spring constant.

(a) \[ k = 57 \text{ N/m} \]

(b) Calculate the displacement, in centimetres, of the same spring when a 0.75 kg mass hangs from it instead.

(b) \[ \Delta x = 13 \text{ cm} \]

October 14, 2012 4U2 - Elastic Potential Energy
Elastic Potential Energy

A graph of Hooke’s law not only gives information about the forces and extensions for a spring (or any elastic substance), you can also use it to determine the quantity of potential energy stored in the spring.

Elastic Potential Energy

As discussed previously, the amount of work done can be determined by finding the area under a force-versus-position graph. A Hooke’s law graph is such a graph, since extension or compression is simply a displacement.

The area under the F-x graph for a spring, therefore, is equal to the amount of elastic potential energy stored in the spring.

$$E_p = \text{area under F-x graph}$$
PRACTICE
4. Find an expression for the area under a F-x graph for a perfectly elastic spring.

\[ E_e = \frac{1}{2} k \Delta x^2 \]

PRACTICE
5. A spring with a spring constant of 75 N/m is resting on a table.
   (a) If the spring is compressed a distance of 28 cm, what is the increase in its potential energy?
   (b) What force must be applied to hold the spring in this position?

(a) \[ \Delta E_e = 2.9 \text{ J} \]
(b) \[ F = 21 \text{ N} \]
Elastic Potential Energy

PRACTICE
6. A 5.3 kg mass hangs vertically from a spring with a spring constant of 720 N/m. The mass is lifted upward and released. Calculate the force and acceleration on the mass when the spring is:
   (a) compressed by 0.36 m.
   (b) stretched by 0.36 m.

   (a) $F = 310 \text{ N \ [down]}$; $a = 59 \text{ m/s}^2 \ [\text{down}]
   
   (b) $F = 210 \text{ N \ [up]}$; $a = 39 \text{ m/s}^2 \ [\text{up}]

Elastic Potential Energy – DYK?

A perfectly elastic material will return precisely to its original form after being deformed. However, no real material is perfectly elastic. Each material has an elastic limit, and when stretched beyond that limit (into the plastic region), the material will not return to its original shape. As a result, the material becomes permanently deformed.

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