2.4 – Simplifying Rational Functions

- GOAL – Define rational functions, and explore methods of simplifying the related rational expression.

Game

- Adonis has designed a game called "2 and 1" to raise money at a charity casino. To start the game, Adonis announces he will draw *n* numbers from a set that includes all the natural numbers from 1 to 2n.
- The players then pick 3 numbers.
- Adonis draws *n* numbers and announces them. The players check for matches. Any player that has at least two matches wins.

The probability of a player winning is given by the **rational function**

$$P(n) = \frac{3n^3 - 3n^2}{8n^3 - 12n^2 + 4n}$$

A "rational function" is a function that is the ratio of two polynomials. f(x) = R(x)/S(x) and S $\neq$ 0

Game cont'd

- For example, if Adonis draws 5 numbers from the set 1 to 10, the probability of winning is:
- $P(5) = \frac{3(5)^3 - 3(5)^2}{8(5)^3 - 12(5)^2 + 4(5)} = \frac{5}{12}$
- The game is played fast, and Adonis needs a fast way to determine the range he should use, based on the number of players and their chances of winning.
- **What is the simplified expression for the probability of a player winning at "2 and 1"?**

Example #1

- You can simplify rational numbers by first factoring numerators and denominators and dividing each by the common factor.
- $P(n) = \frac{3n^3 - 3n^2}{8n^3 - 12n^2 + 4n}$
- $= \frac{3n^2(n-1)}{4n(2n^2-3n+1)}$
- $= \frac{3n^2(n-1)}{4n(2n-1)(n-1)}$
- $= \frac{3n^2}{4n(2n-1)}$

Because this is a **rational function (fraction)**, the denominator can never equal 0.

RESTRICTIONS:

When $4n(2n - 1)(n - 1) = 0$

$n = 0$, $n = ½$, $n = 1$

Therefore:

$$P(n) = \frac{3n^2}{4n(2n-1)}; n \neq 0, \frac{1}{2}, 1$$

Example #2

- Simplify and state any restrictions on the variables.
- $\frac{30x^4y^3}{-6x^7y}$

Factor out the numerator and denominator by factoring out the GCF.

- $= \frac{\cancel{-6x^4y}(-5y^2)}{\cancel{-6x^4y}(x^3)}$
- $= \frac{-5y^2}{x^3}; x, y \neq 0$
- Always determine the restrictions using the zeros of the **original** denominator:
- $-6x^4y(x^3)$, so $x, y \neq 0$.

Example #3

- Simplify and state any restrictions on the variables.
- $\frac{10x^4-8x^2+4x}{2x^2}$

Factor out the numerator and denominator by factoring out the GCF.

- $= \frac{2x(5x^3-4x+2)}{2x(x)}$
- $= \frac{5x^3-4x+2}{x}; x \neq 0$

Example #4

- Simplify f(x) and state the domain, where $f(x) = \frac{x^2+7x-8}{2-2x}$.
- $f(x) = \frac{x^2+7x-8}{2-2x}$
- $= \frac{(x-1)(x+8)}{2(1-x)}$
- $= \frac{-(1-x)(x+8)}{2(1-x)}$
- $= \frac{-(x+8)}{2}, x \neq 1$
- Therefore the domain is D = $\{ x \neq 1 \mid x \in \mathrm{R}\}$

In Summary…

- A rational **function** can be expressed as the ratio of two polynomial functions. For example:
- $f(x) = \frac{6x+2}{x-1}, x \neq 1$
- A rational **expression** is the ratio of two polynomials. For example:
- $\frac{6x+2}{x-1}, x \neq 1$
- Both the functions and expressions are **undefined** for numbers that make the denominator zero.