2.7 – Adding & Subtracting Rational Expressions

- GOAL – Develop strategies for adding and subtracting rational expressions.

A jet flies along a straight path from Toronto to Montreal and back again. The straight-line distance between these cities is 540 km. On Monday, the jet made the round trip when there was no wind. On Friday, it made the round trip when there was a constant wind blowing from Toronto to Montreal at 80 km/h. While travelling in still air, the jet travels at constant speed.

- Which round trip takes less time?

Example #1 cont'd

- v is the jet's airspeed in still air.
- $v + 80$ is the jet's airspeed from Toronto to Montreal.
- $v - 80$ is the jet's airspeed from Montreal to Toronto.
- $\frac{540}{v}$ is the time elapsed when there is no wind.
- $\frac{540}{v+80}$ is the time elapsed from Toronto to Montreal.
- $\frac{540}{v-80}$ is the time elapsed from Montreal to Toronto.

Example #1 cont'd

- Time for Round Trip: No Wind:
- $T_1 = \frac{540}{v} + \frac{540}{v} = \frac{1080}{v}$
- Time for Round Trip: With Wind:
- $T_2 = \frac{540}{v+80} + \frac{540}{v-80}$
- $= \frac{540(v-80) + 540(v+80)}{(v+80)(v-80)}$
- $= \frac{1080v}{v^2-6400}$

$T_1 = \frac{1080}{v} \times \frac{v}{v}$

$= \frac{1080v}{v^2}$

Therefore the trip without wind took less time.

Example #2

- Simplify and state any restrictions on the variables: $\frac{3}{8x^2} + \frac{1}{4x} - \frac{5}{6x^3}$
- Lowest common denominator (LCD) = $24x^3$
- $\frac{3}{8x^2} + \frac{1}{4x} - \frac{5}{6x^3} = \frac{(3x)3}{(3x)8x^2} + \frac{(6x^2)1}{(6x^2)4x} - \frac{(4)5}{(4)6x^3}$
- $= \frac{9x+6x^2-20}{24x^3}, x \neq 0$

Example #3

- Simplify and state any restrictions on the variables: $\frac{3n}{2n+1} + \frac{4}{n-3}$

$$\text{LCD} = (2n+1)(n-3)$$

$$\frac{3n}{2n+1} + \frac{4}{n-3}$$

$$= \frac{(n-3)3n}{(2n+1)(n-3)} + \frac{(2n+1)4}{(2n+1)(n-3)}$$

Example #3 cont'd

$$= \frac{(n-3)3n}{(2n+1)(n-3)} + \frac{(2n+1)4}{(2n+1)(n-3)}$$

$$= \frac{(n-3)3n + (2n+1)4}{(2n+1)(n-3)}$$

$$= \frac{3n^2 - 9n + 8n + 4}{(2n+1)(n-3)}$$

$$= \frac{3n^2 - n + 4}{(2n+1)(n-3)}; \; x \neq -\frac{1}{2}, 3$$

Example #4

- Simplify and state restrictions on the variables: $\frac{2t}{t^2-1} - \frac{t+2}{t^2+3t-4}$

$$= \frac{2t}{(t-1)(t+1)} - \frac{t+2}{(t+4)(t-1)}$$

$$= \frac{(t+4)2t}{(t-1)(t+1)(t+4)} - \frac{(t+1)(t+2)}{(t+1)(t-1)(t+4)}$$

$$= \frac{2t^2+8t-t^2-2t-t-2}{(t-1)(t+1)(t+4)}$$

$$= \frac{t^2+5t-2}{(t-1)(t+1)(t+4)}; t \neq 1, -1, -4$$

In Summary…

- The procedures for adding and subtracting rational functions are the same as those for adding and subtracting rational numbers. When rational expressions are added or subtracted, they must have a common denominator.