3.2 – Determining Max and Min Values of a Quadratic Function

- GOAL – Use a variety of strategies to determine the maximum or minimum value of a quadratic function.
- A golfer attempts to hit a golf ball over a gorge from a platform above the ground. The function that models the height of the ball is $h(t) = -5t^2 + 40t + 100$, where $h(t)$ is the height in meters at time t seconds after contact. There are power lines 185m above the ground.
- **Will the golf ball hit the power lines?**

Solution

- We want to see if the max height is higher or lower than the height of the power lines. We can complete the square:
- $h(t) = -5t^2 + 40t + 100$
- $h(t) = -5(t^2 – 8t) + 100$
- $\quad = -5(t^2 – 8t + 16) + 100 + 80$
- $\quad = -5(t – 4)^2 + 180$

- Thus, the vertex is at (4, 180). The maximum height will be 180m after 4 seconds.
- Since the power lines are 185 m above the ground, the ball will not hit them.

Example #2

- The demand function for a new magazine is p(x) = -6x + 40, where p(x) represents the selling price, in thousands of dollars, of the magazine and *x* is the number sold, in thousands. The cost function is C(x) = 4x + 48. Calculate the maximum profit and the number of magazines sold that will produce the maximum profit.

- Revenue = Demand x Number Sold
- = [p(x)] (x)

- Profit = Revenue – Cost
- = [p(x)] (x) – C(x)

Example #2 cont'd

Profit = Revenue – Cost

- = [p(x)] (x) – C(x)
- = (-6x + 40)(x) – (4x + 48)
- = $-6x^2 + 40x - 4x - 48$
- = $-6x^2 + 36x - 48$
- We can find the zeros, and the x-value in their centre is the number of magazines to give the maximum profit.
- = (-3x + 6)(2x - 8) = -3(x - 2)(2)(x - 4)
- x = 2 and x = 4 - In the middle, we have x = 3 (3000 magazines).
- $P(3) = -6(3)^2 + 36(3) - 48 = -54 + 108 - 48 = 6$
- Therefore, the maximum profit is $6000 when 3000 magazines are sold.