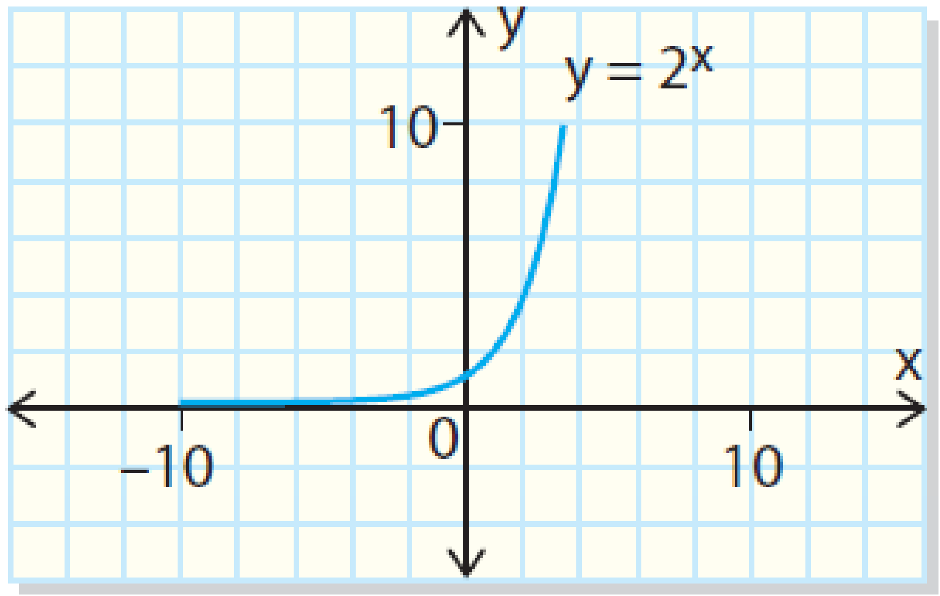
4.6 - Transformations of Exponential Functions

- GOAL – Investigate the effects of transformations on the graphs and equations of exponential functions.



This is the graph of the function $f(x) = 2^x$.

- ❖ It is an increasing function
- ❖ It has a y-intercept of 1
- ❖ Its asymptote is the line $y = 0$

If we go from $f(x) = 2^x$ to $g(x) = af(k(x – d)) + c$, what happens to the size and shape of $f(x)$?

Function	Sketch	Table of Values	Description of Changes of New Graph
$g(x) = 2^x + 1$			
$h(x) = 2^x - 1$			
$p(x) = 2^{x+1}$			
$q(x) = 2^{x-1}$			

Table of Values for $g(x) = 2^x + 1$:

x	$y = 2^x$	$y = 2^x + 1$
-1		
0		
1		

Table of Values for $h(x) = 2^x - 1$:

x	$y = 2^x$	$y = 2^x - 1$
-1		
0		
1		

Table of Values for $p(x) = 2^{x+1}$:

x	$y = 2^x$	$y = 2^{x+1}$
-1		
0		
1		

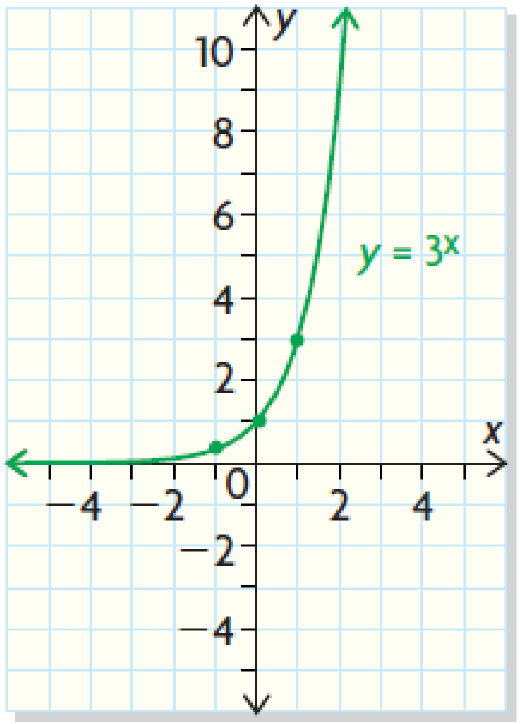
Table of Values for $q(x) = 2^{x-1}$:

x	$y = 2^x$	$y = 2^{x-1}$
-1		
0		
1		

Take a few minutes and fill in the table on the left.

Example #1

- Use transformations to sketch $y = -2(3^{x-4})$. State the domain and range.



3 key points: (0, 1), (1, 3), (-1, 1/3)

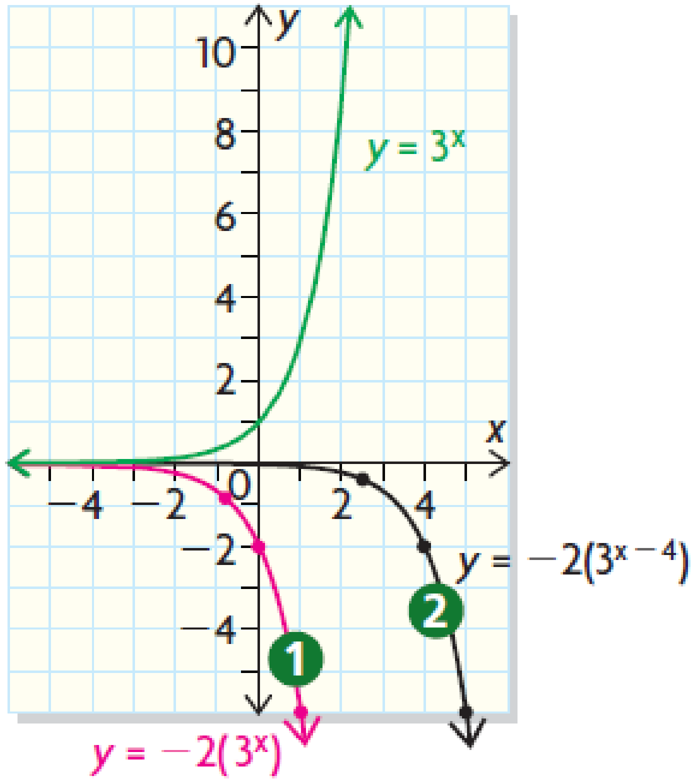
Asymptote is x-axis: y = 0

1. $y = -2(3^x)$:

- *Vertical stretch by factor 2 and*
- *Reflection over the x-axis*

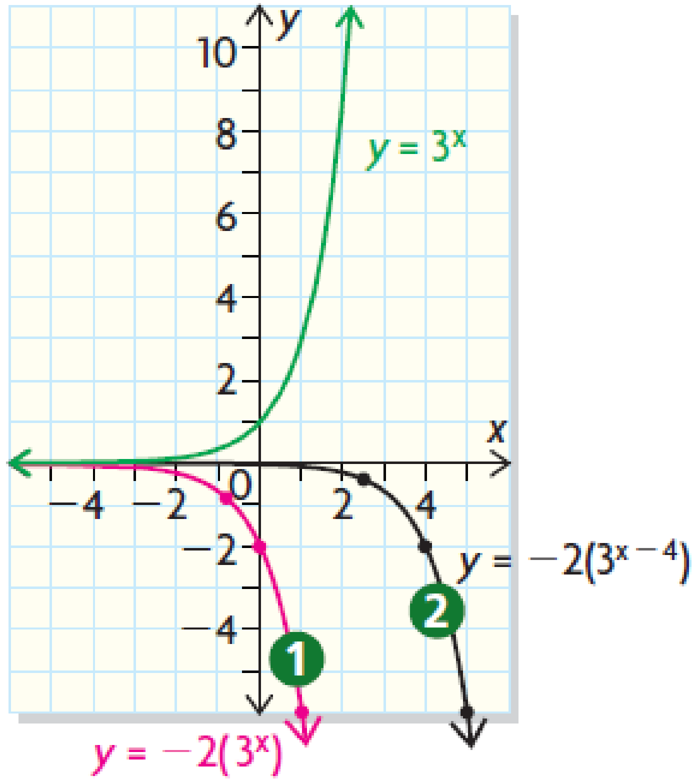
2. $y = -2(3^{x-4})$:

- *Horizontal shift right 4 units*



Example #1 cont'd

- Use transformations to sketch $y = -2(3^{x-4})$. State the domain and range.



Original Function

$D = \{ x \in R \}$

$R = \{ y > 0 \mid y \in R \}$

Transformed Function

$D = \{ x \in R \}$

$R = \{ y < 0 \mid y \in R \}$

Example #2

- Compare and contrast the functions defined by $f(x) = 9^x$ and $g(x) = 3^{2x}$.
- ***Use exponent rules***

- **$f(x) = 9^x$**
- **$= (3^2)^x$**
- **$= 3^{2x}$**
- **$= g(x)$**

- **Therefore, both functions are the same.**

Example #3

- An exponential function with a base of 2 has been stretched vertically by a factor 1.5 and reflected over the y-axis. Its asymptote is the line y = 2. Its y-intercept is (0, 3.5). Write an equation of the function and discuss its domain and range.
- We currently have something like this:
- $y = a*2^{k(x-d)} + c$

D = $\{ x \in R \}$ – DIDN'T CHANGE.
RANGE WAS: $R = \{ y > 0 \mid y \in R \}$
RANGE IS NOW: $R = \{ y > 2 \mid y \in R \}$

- Stretch vertically by factor 1.5: **a = 1.5**
- Reflected over the y-axis: **x becomes –x**
- Asymptote is the line y = 2: Horizontal asymptote moved from y = 0 to y = 2: **c =2**

- Therefore, **$y = 1.5(2^{-x}) + 2$**. To check that we have the right equation, just plug in x = 0 and confirm that you get y = 3.5 as the answer.