7.2 - Geometric Sequences

- GOAL – Recognize the characteristics of geometric sequences and express the general terms in a variety of ways.

EX. 1: a) Determine the 13th term of a geometric sequence if the first term is 9 and the common ratio is 2.

n	1	2	3	4	5	6	7
t_n	9	18	36	72	144	288	576

n	8	9	10	11	12	13
t_n	1152	2304	4608	9216	18 432	36 864

A **geometric sequence** because the terms increase by the same **multiple** each time (i.e. x 2)

The 13th term is 36 864.

Example #1 cont'd

- B) State a formula that defines each term of any geometric sequence.

n	1	2	3	4	5	6	7
t_n	9	18	36	72	144	288	576

n	8	9	10	11	12	13
t_n	1152	2304	4608	9216	18 432	36 864

The pattern here is:

$a, ar, (ar)r, (ar^2)r, \ldots$

The recursive formula is $t_1 = a$. $\boldsymbol{t_n = rt_{n-1,}}$ where $n \in \mathbf{N}$ and $n > 1$.

Example #2

A company has 3 kg of radioactive material that must be stored until it becomes safe to the environment. After one year, 95% of the radioactive material remains. How much radioactive material will be left after 100 years?

$3, 3 \times 0.95, (3 \times 0.95) \times 0.95, (3 \times 0.95^2) \times 0.95, \ldots$

$= 3, 3 \times 0.95, 3 \times 0.95^2, 3 \times 0.95^3, \ldots$

$a = 3$

$r = 0.95$

$f(n) = ar^{n-1}$

$f(100) = 3 \times 0.95^{99}$

$\tilde{=} 0.019$

After the 100^{th} year, there will be about 19g of radioactive material left.

Example #3

- How many terms are in the geometric sequence 52 612 659, 17 537 553, … , 11?
- a = 52 612 659
- $r = \frac{17\ 537\ 553}{52\ 612\ 659} = \frac{1}{3}$
- $f(n) = ar^{n-1}$
- $11 = 52\ 612\ 659 \times \left(\frac{1}{3}\right)^{n-1}$
- $2.1 \times 10^{-7} = \left(\frac{1}{3}\right)^{n} \left(\frac{1}{3}\right)^{-1}$

This is equal to 3

- $7 \times 10^{-8} = \left(\frac{1}{3}\right)^{n}$

n is around 16.5

In Summary…

- A geometric sequence can be defined
  - by the general term $t_n = ar^{n-1}$,
  - recursively by $t_1 = a$, $t_n = rt_{n-1}$, where $n > 1$, or
  - by a discrete exponential function $f(n) = ar^{n-1}$.

  In all cases, $n \in \mathbf{N}$, a is the first term, and r is the common ratio.