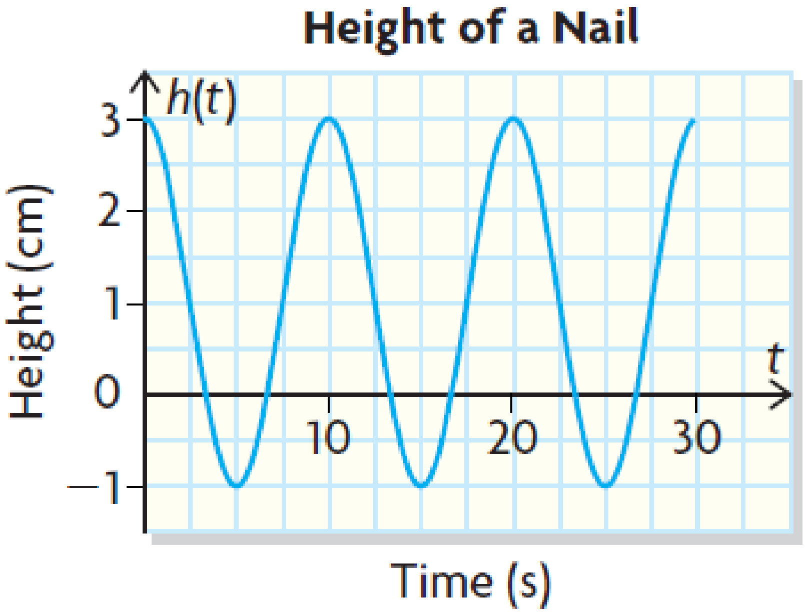
6.6 – Investigating Models of Sinusoidal Functions

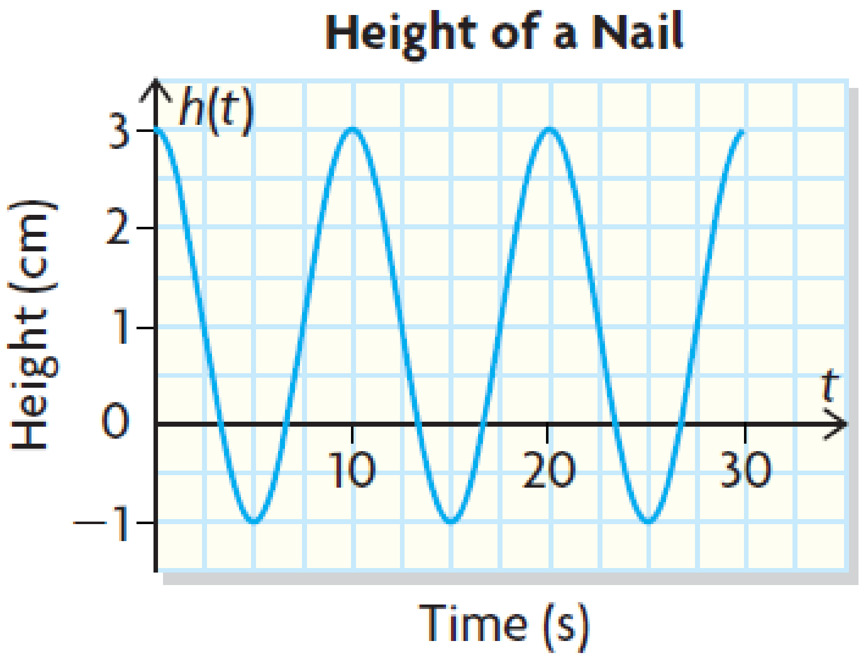
- GOAL – Determine the equation of a sinusoidal function from a graph or a table of values.

A nail located on the circumference of a water wheel is moving as the current pushes the wheel. The height of the nail in terms of time can be modelled by the graph shown.



How can you determine the equation of a sinusoidal function from its graph?

Example #1 cont'



- **Horizontal compression factor: *k***
- **Period =** $\frac{360}{|k|}$
- The period is 10s, so |k| = 360 / 10 = 36
- Therefore, all the x-values were multiplied/compressed by $\frac{1}{|k|} = \frac{1}{36}$
- **Vertical translation = c**
- Equation of the axis: h = 1 therefore c = 1
- **Vertical stretch = *a***
- Since amplitude is 4, *a* = 2

Base graph: h(t) = cost

With all the previously mentioned transformations, we end up with:

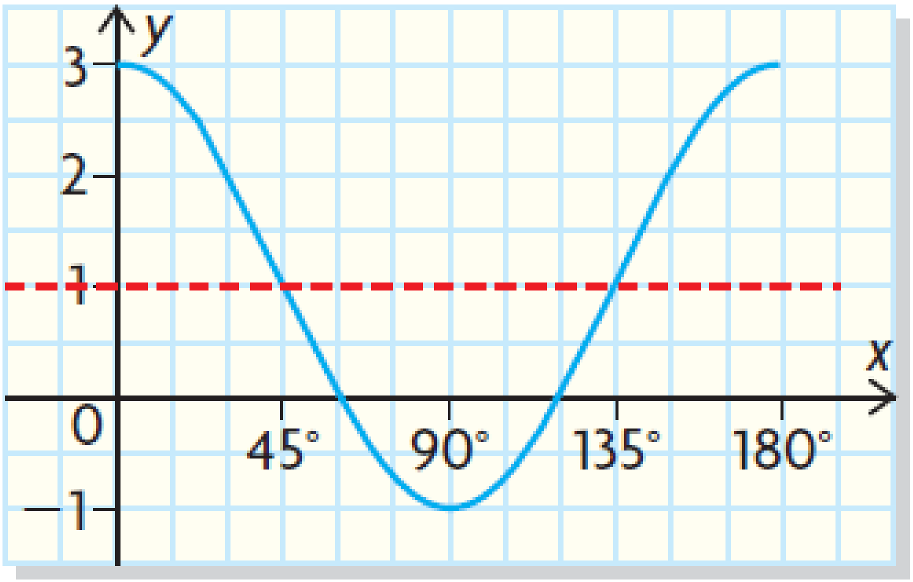
h(t) = 2cos(36t) + 1

As a sine curve (shift cosine 7.5s to the right),

h(t) = 2sin(36(t - 7.5)) + 1

Example #2

- A sinusoidal function has an amplitude of 2 units, a period of 180º, and a maximum at (0, 3). Represent the function with an equation in two different ways.



- Vertical translation: c = 1
- Vertical stretch: *a*; amplitude = (0.5)(3 – (-1)) = 2
- Horizontal compression: *k*
  - Period = $\frac{360^o}{k}$
  - $180^o = \frac{360^o}{k}$
  - $k = \frac{360^o}{180^o}$
  - $k = 2$, so the compression factor is ½

- For a cosine curve, there is no horizontal translation (phase shift) so *d* = 0.
- Equation: $y = 2\cos(2x) + 1$
- For a sine curve, horizontal translation is 135º (270º / 2)
- Equation: $y = 2\sin\left(2(\mathrm{x} - 135^o)\right) + 1$

In Summary…

- If you are given a set of data, and the corresponding graph is a sinusoidal function, you can determine the equation by calculating the graph’s period, amplitude, and equation of the axis.