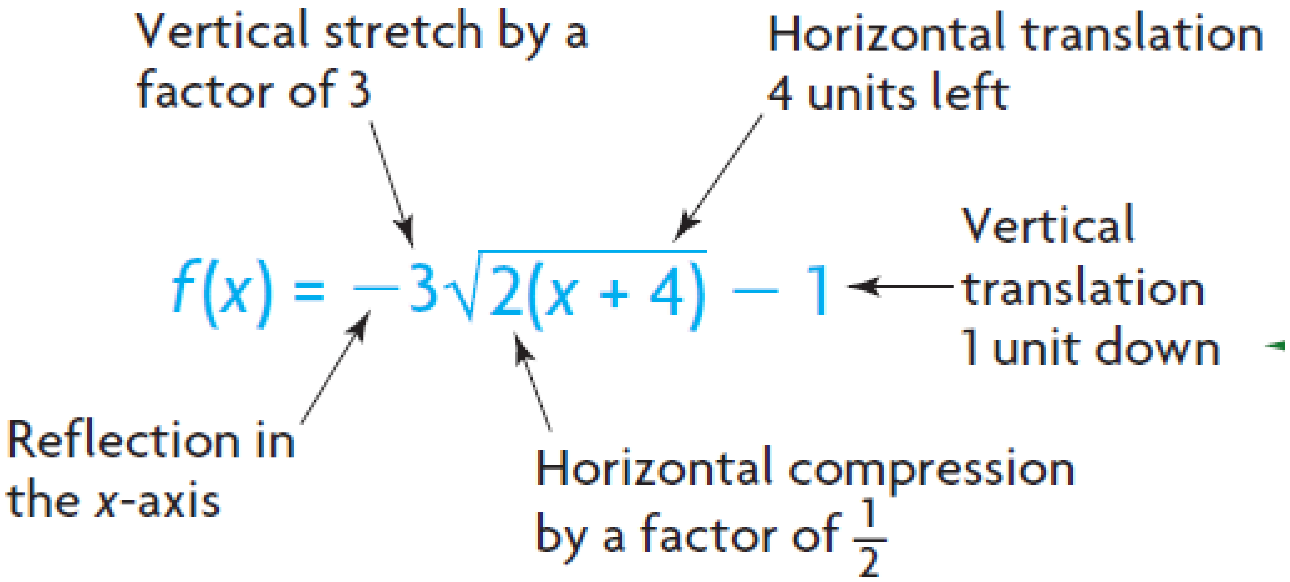
1.8 - Using Transformations to Graph Functions of the Form y = af[k (x - d)] + c

- GOAL: Apply combinations of transformations, in a systematic order, to sketch graphs of functions.
- Neil wants to sketch the graph of $f(x) = -3\sqrt{2(x+4)} - 1$.
- How can Neil apply the transformations necessary to sketch the graph?

Example #1

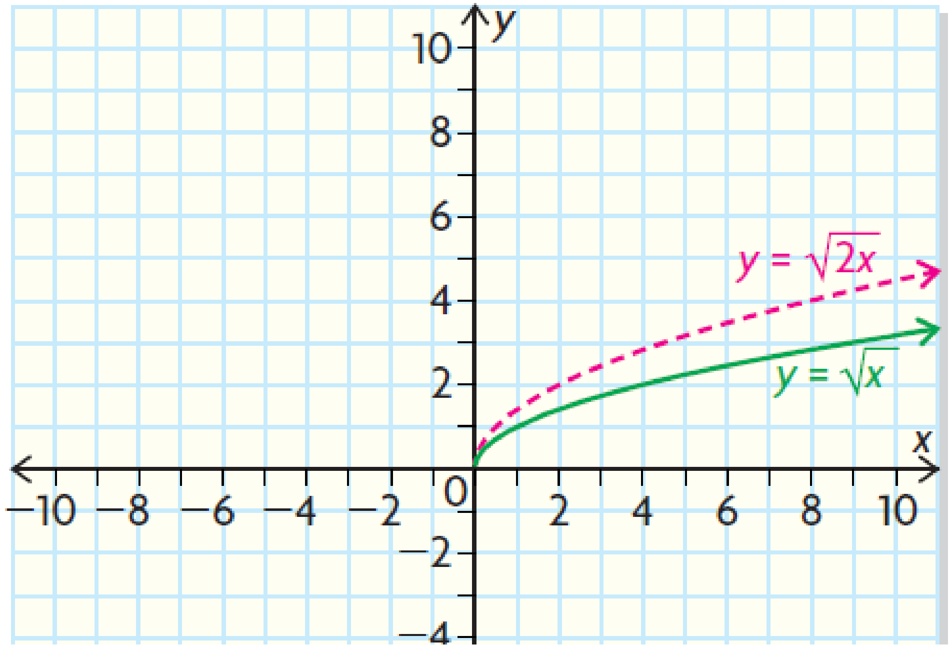
- Sketch the graph of $f(x) = -3\sqrt{2(x+4)} - 1$. State the domain and range of the transformed function.
- The parent function is $f(x) = \sqrt{x}$.



Example #1 cont'd

- Let's apply 1 transformation at a time to the parent function until we get the transformed function.
- 1. Let's graph $f(x) = \sqrt{x}$ and then $f(x) = \sqrt{2x}$. Divide the x coordinates of points on $y = \sqrt{x}$ to compress the graph horizontally by the factor ½ .

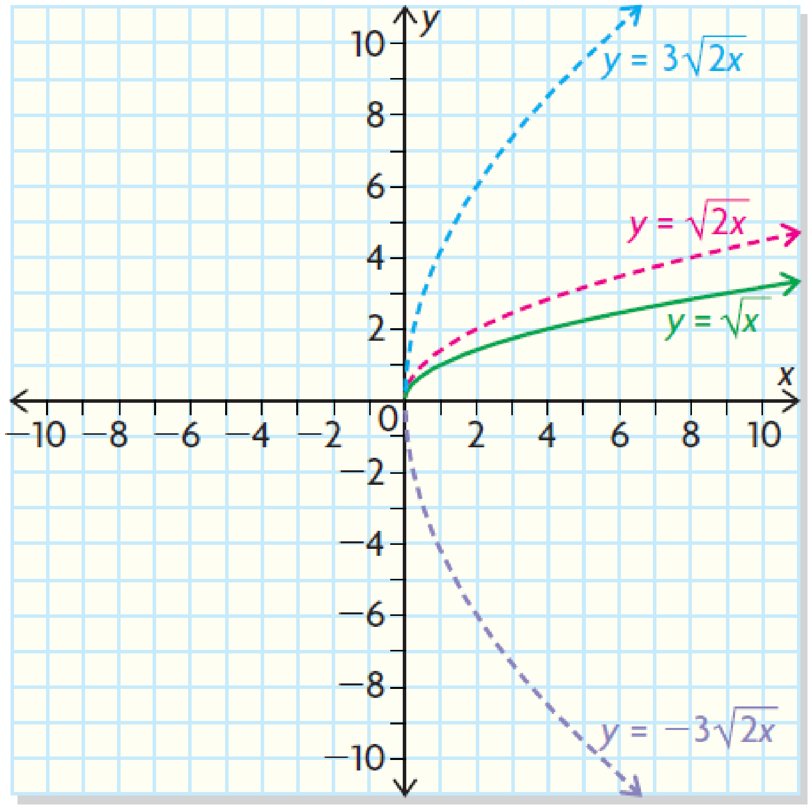
$f(x)$	$f(2x)$
(0, 0)	(0, 0)
(1, 1)	$\left(\frac{1}{2}, 1\right)$
(4, 2)	(2, 2)
(9, 3)	$\left(4\frac{1}{2}, 3\right)$



Example #1 cont'd

- 3. Let's graph $f(x) = -3\sqrt{2x}$ using $f(x) = 3\sqrt{2x}$. Flip the

$f(x)$	$f(2x)$	$3f(2x)$	$-3f(2x)$
(0, 0)	(0, 0)	(0, 0)	(0, 0)
(1, 1)	$\left(\frac{1}{2}, 1\right)$	$\left(\frac{1}{2}, 3\right)$	$\left(\frac{1}{2}, -3\right)$
(4, 2)	(2, 2)	(2, 6)	(2, −6)
(9, 3)	$\left(4\frac{1}{2}, 3\right)$	$\left(4\frac{1}{2}, 9\right)$	$\left(4\frac{1}{2}, -9\right)$



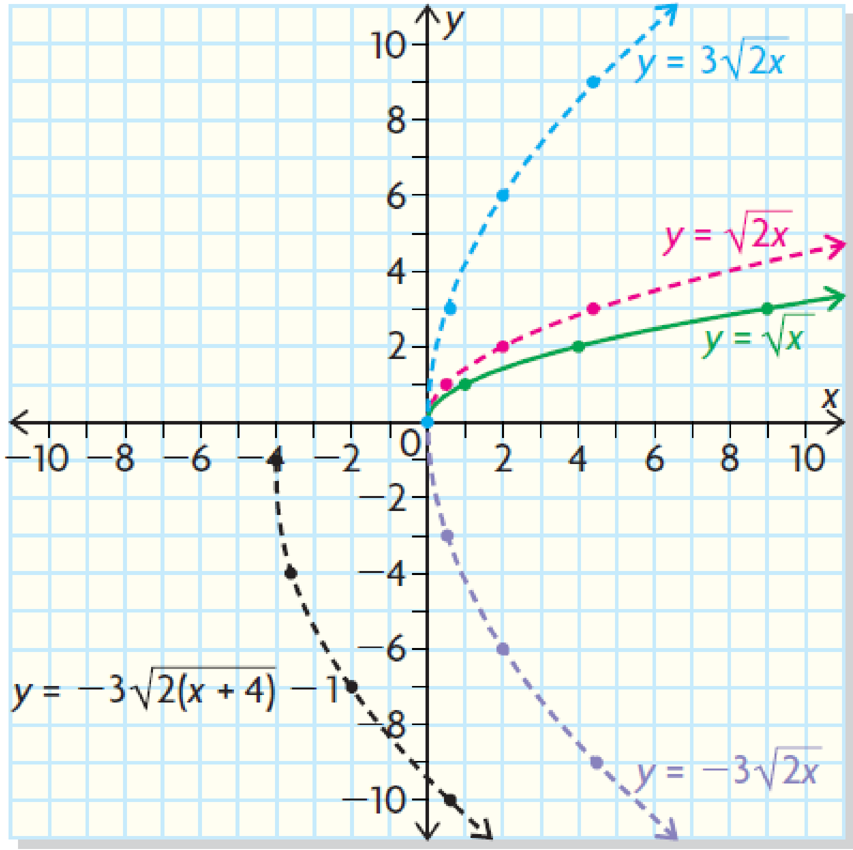
Example #1 cont'd

DOMAIN = { x ≥ -4 | x ∈ **R**}

RANGE = { y ≤ -1 | y ∈ **R**}

- 4. Now we simply translate the graph of $f(x) = -3\sqrt{2x}$ 4 units left and 1 unit down.

$f(x)$	$f(2x)$	$3f(2x)$	$-3f(2x)$	$-3f(2(x+4))-1$
$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(-4, -1)$
$(1, 1)$	$\left(\frac{1}{2}, 1\right)$	$\left(\frac{1}{2}, 3\right)$	$\left(\frac{1}{2}, -3\right)$	$\left(-3\frac{1}{2}, -4\right)$
$(4, 2)$	$(2, 2)$	$(2, 6)$	$(2, -6)$	$(-2, -7)$
$(9, 3)$	$\left(4\frac{1}{2}, 3\right)$	$\left(4\frac{1}{2}, 9\right)$	$\left(4\frac{1}{2}, -9\right)$	$\left(\frac{1}{2}, -10\right)$



Example #1 cont'd

- Let's apply 1 transformation at a time to the parent function until we get the transformed function.
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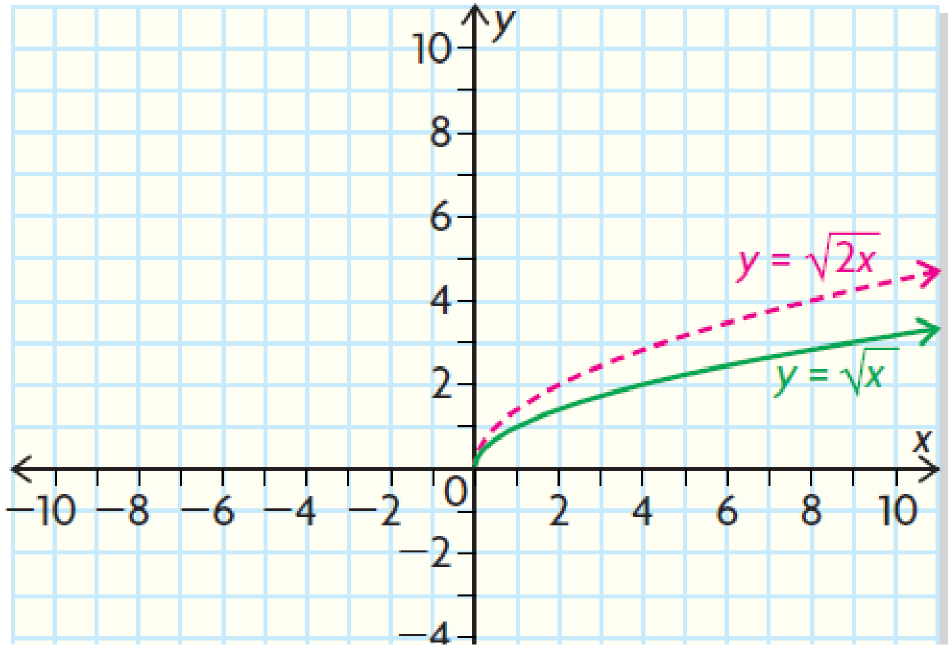


Example #2

- Some transformations are applied, in order, to the reciprocal function $f(x) = \frac{1}{x}$:
  - horizontal stretch by a factor of 3
  - vertical stretch by a factor of 2
  - reflection in the y-axis
  - translation 5 units right and 4 units up

a) Write the equation for the final transformed function g(x).

b) Sketch the graphs of f(x) and g(x).

c) State the domain and range of both functions.

Example #2 (A)

- $f(x) = \frac{1}{x}$:
  - horizontal stretch by a factor of 3
  - vertical stretch by a factor of 2
  - reflection in the y-axis
  - translation 5 units right and 4 units up

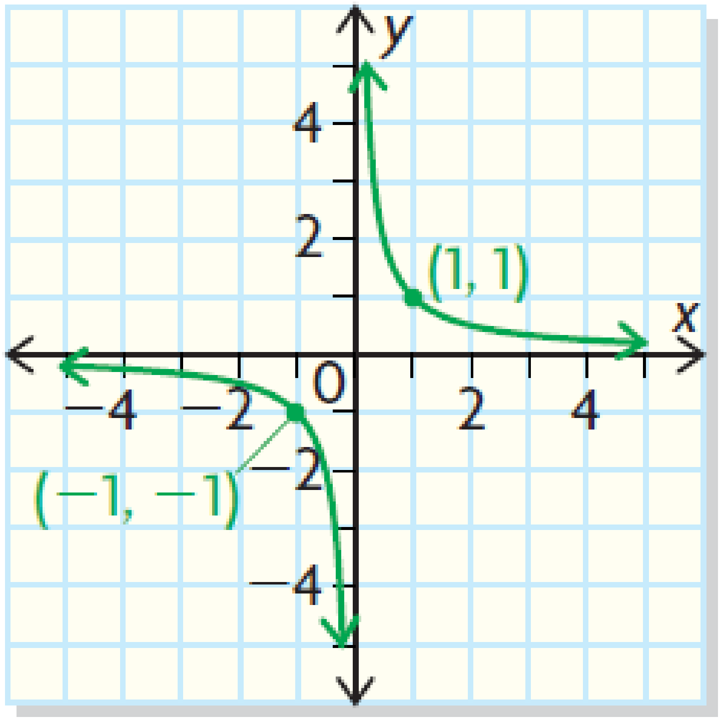
A) $g(x) = af[k(x - d)] + c$

a = 2 (vertical stretch); *d* = 5 and *c* = 4 because the translation is 5 units right and 4 units up; *k* = -1/3 due to a horizontal stretch by the factor 3 and a reflection in the y-axis

$$g(x) = 2f\left[\frac{-1}{3}(x - 5)\right] + 4$$

Example #2 (B)

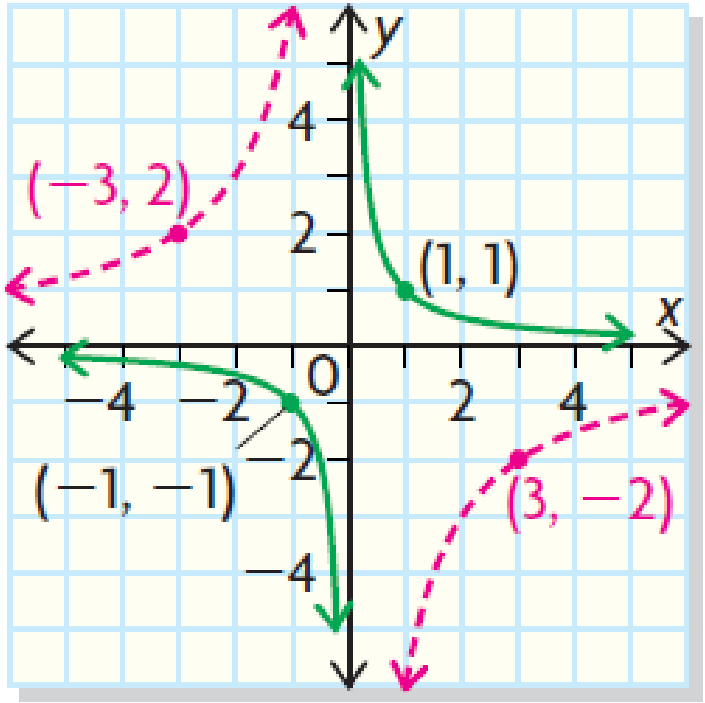
Sketch the graph of f(x). The points (1, 1) and (-1, -1) have been labeled. The vertical asymptote is x = 0 and the horizontal asymptote is y = 0.



1. Apply the stretches and reflection to the labeled points – multiply x-coordinates by -3 and y-coordinates by 2.

(1, 1) becomes (-3, 2) and (-1, -1) becomes (3, -2).

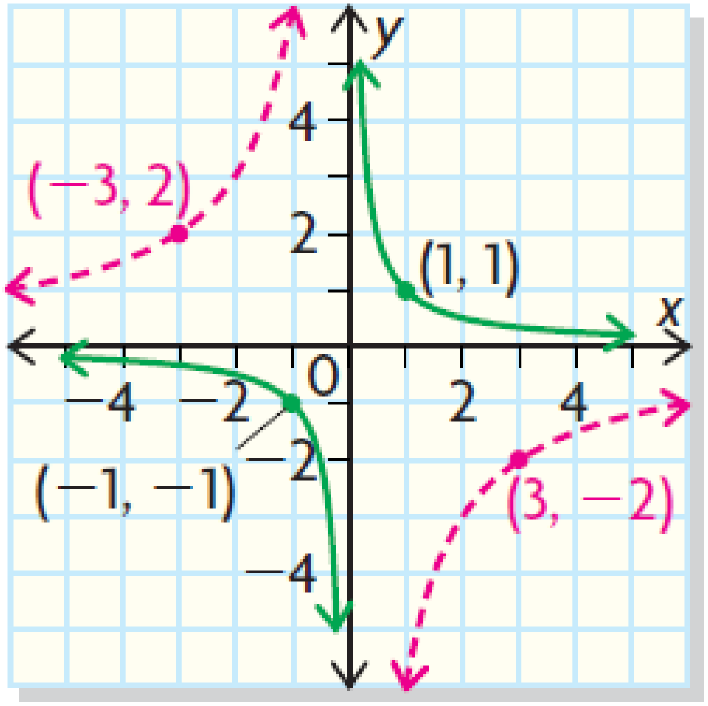
*The asymptotes didn't change.



Example #2 (B)

To apply the translations, 5 right and 4 up, I drew in the translated asymptotes first.

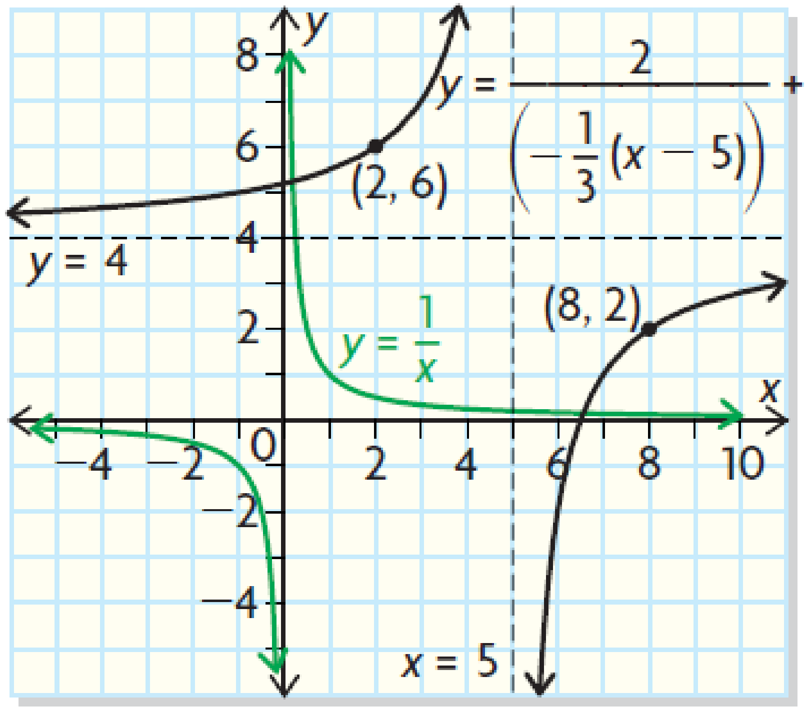
New vertical asymptote is at x = 5; New horizontal asymptote is at y = 4



Simply draw the stretched and reflected graph in the new position after the translation.

For f(x):
DOMAIN = { $x \neq 0$ | $x \in$ **R** }
RANGE = { $y \neq 0$ | $y \in$ **R** }

For g(x):
DOMAIN = { $x \neq 5$ | $x \in$ **R** }
RANGE = { $y \neq 4$ | $y \in$ **R** }



Example #4

Match each equation to its graph. Explain your reasoning.

1. $y = \dfrac{1}{0.3(x+1)} - 2$

2. $y = -4|x+2| + 1$

3. $y = -\sqrt{3(x+2)} - 1$

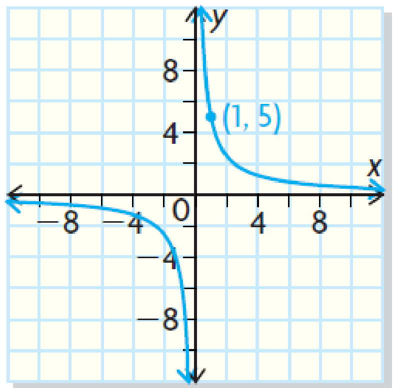
4. $y = \sqrt{-0.4(x-4)} + 3$

5. $y = (0.5(x-4))^2 + 2$

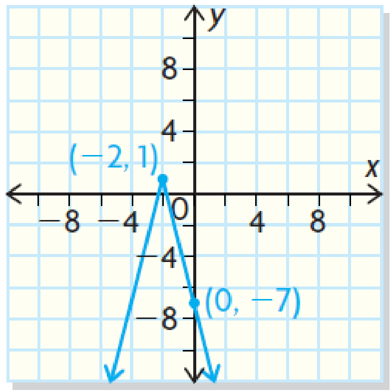
6. $y = \dfrac{5}{x}$

7. $y = -3(x+1)^2 + 4$

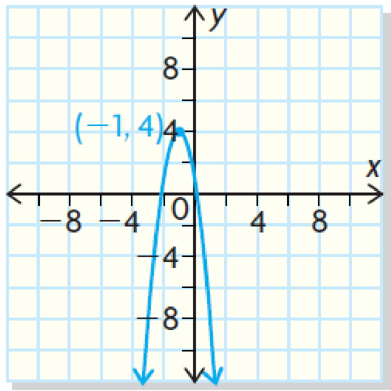
A



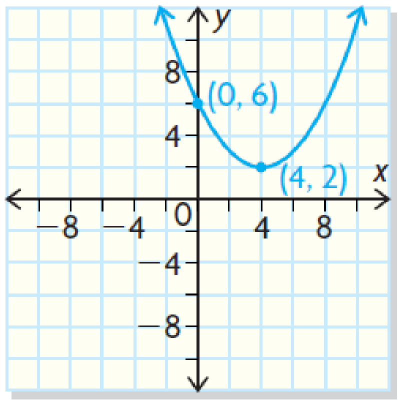
D



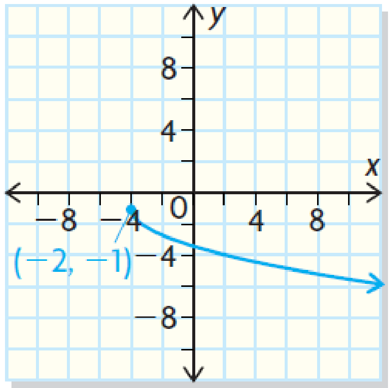
G



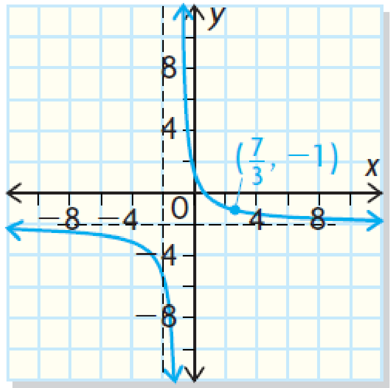
C



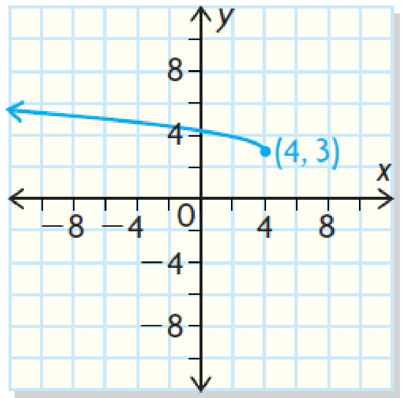
B



E



F



A : 6
B : 3
C : 5
D : 2
E : 1
F : 4
G : 7