

LINEAR EQUATIONS*

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Abstract

This chapter covers principles of linear equations. After completing this chapter students should be able to: graph a linear equation; find the slope of a line; determine an equation of a line; solve linear systems; and complete application problems using linear equations.

1 Chapter Overview

In this chapter, you will learn to:

1. Graph a linear equation.
2. Find the slope of a line.
3. Determine an equation of a line.
4. Solve linear systems.
5. Do application problems using linear equations.

2 Graphing a Linear Equation

Equations whose graphs are straight lines are called **linear equations**. The following are some examples of linear equations:

$$2x - 3y = 6, 3x = 4y - 7, y = 2x - 5, 2y = 3, \text{ and } x - 2 = 0.$$

A line is completely determined by two points, therefore, to graph a linear equation, we need to find the coordinates of two points. This can be accomplished by choosing an arbitrary value for x or y and then solving for the other variable.

Example 1

Graph the line: $y = 3x + 2$

Solution

We need to find the coordinates of at least two points.

We arbitrarily choose $x = -1$, $x = 0$, and $x = 1$.

If $x = -1$, then $y = 3(-1) + 2$ or -1 . Therefore, $(-1, -1)$ is a point on this line.

If $x = 0$, then $y = 3(0) + 2$ or $y = 2$. Hence the point $(0, 2)$.

If $x = 1$, then $y = 5$, and we get the point $(1, 5)$. Below, the results are summarized, and the line is graphed.

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X	-1	0	1
Y	-1	2	5

Table 1

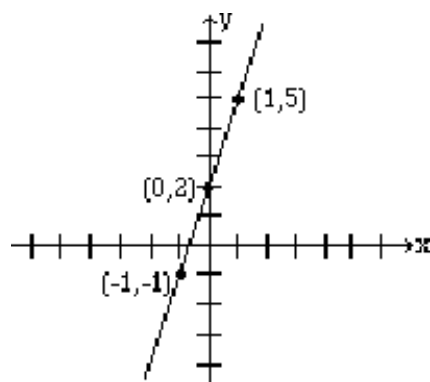


Figure 1

Example 2

Graph the line: $2x + y = 4$

Solution

Again, we need to find coordinates of at least two points.

We arbitrarily choose $x = -1$, $x = 0$ and $y = 2$.

If $x = -1$, then $2(-1) + y = 4$ which results in $y = 6$. Therefore, $(-1, 6)$ is a point on this line.

If $x = 0$, then $2(0) + y = 4$, which results in $y = 4$. Hence the point $(0, 4)$.

If $y = 2$, then $2x + 2 = 4$, which yields $x = 1$, and gives the point $(1, 2)$. The table below shows the points, and the line is graphed.

x	-1	0	1
y	6	4	2

Table 2

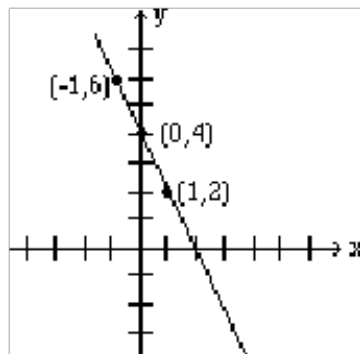


Figure 2

The points at which a line crosses the coordinate axes are called the **intercepts**. When graphing a line, intercepts are preferred because they are easy to find. In order to find the x-intercept, we let $y = 0$, and to find the y-intercept, we let $x = 0$.

Example 3

Find the intercepts of the line: $2x - 3y = 6$, and graph.

Solution

To find the x-intercept, we let $y = 0$ in our equation, and solve for x .

$$2x - 3(0) = 6 \quad (1)$$

$$2x - 0 = 6 \quad (2)$$

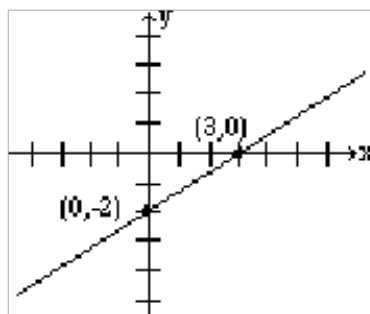
$$2x = 6 \quad (3)$$

$$x = 3 \quad (4)$$

Therefore, the x-intercept is 3.

Similarly by letting $x = 0$, we obtain the y-intercept which is -2.

NOTE: If the x-intercept is 3, and the y-intercept is -2, then the corresponding points are (3, 0) and (0, -2), respectively.

**Figure 3**

In higher math, equations of lines are sometimes written in parametric form. For example, $x = 3 + 2t$, $y = 1 + t$. The letter t is called the parameter or the dummy variable. Parametric lines can be graphed by finding values for x and y by substituting numerical values for t .

Example 4

Graph the line given by the parametric equations: $x = 3 + 2t$, $y = 1 + t$

Solution

Let $t = 0, 1$ and 2 , and then for each value of t find the corresponding values for x and y .

The results are given in the table below.

t	0	1	2
x	3	5	7
y	1	2	3

Table 3

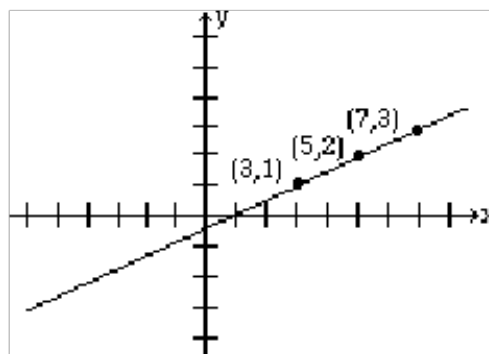


Figure 4

2.1 Horizontal and Vertical Lines

When an equation of a line has only one variable, the resulting graph is a horizontal or a vertical line.

The graph of the line $x = a$, where a is a constant, is a vertical line that passes through the point $(a, 0)$. Every point on this line has the x-coordinate a , regardless of the y-coordinate.

The graph of the line $y = b$, where b is a constant, is a horizontal line that passes through the point $(0, b)$. Every point on this line has the y-coordinate b , regardless of the x-coordinate.

Example 5

Graph the lines: $x = -2$, and $y = 3$.

Solution

The graph of the line $x = -2$ is a vertical line that has the x-coordinate -2 no matter what the y-coordinate is. Therefore, the graph is a vertical line passing through $(-2, 0)$.

The graph of the line $y = 3$, is a horizontal line that has the y-coordinate 3 regardless of what the x-coordinate is. Therefore, the graph is a horizontal line that passes through $(0, 3)$.

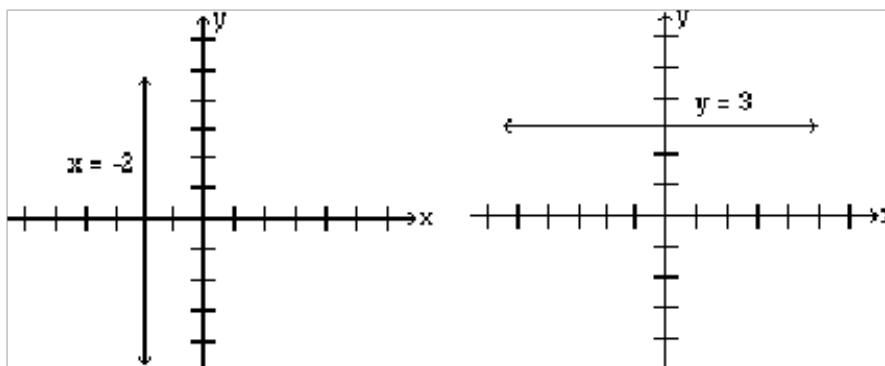


Figure 5

NOTE: Most students feel that the coordinates of points must always be integers. This is not true, and in real life situations, not always possible. Do not be intimidated if your points include numbers that are fractions or decimals.

3 Slope of a Line

3.1 Section Overview

In this section, you will learn to:

1. Find the slope of a line if two points are given.
2. Graph the line if a point and the slope are given.
3. Find the slope of the line that is written in the form $y = mx + b$.
4. Find the slope of the line that is written in the form $Ax + By = c$.

In the last section, we learned to graph a line by choosing two points on the line. A graph of a line can also be determined if one point and the "steepness" of the line is known. The precise number that refers to the steepness or inclination of a line is called the **slope** of the line.

From previous math courses, many of you remember slope as the "rise over run," or "the vertical change over the horizontal change" and have often seen it expressed as:

$$\frac{\text{rise}}{\text{run}}, \frac{\text{vertical change}}{\text{horizontal change}}, \frac{\Delta y}{\Delta x} \text{ etc.} \quad (5)$$

We give a precise definition.

Definition 1:

If (x_1, y_1) and (x_2, y_2) are two different points on a line, then the slope of the line is

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 6

Find the slope of the line that passes through the points $(-2, 3)$ and $(4, -1)$, and graph the line.

Solution

Let $(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (4, -1)$ then the slope

$$m = \frac{-1 - 3}{4 - (-2)} = -\frac{4}{6} = -\frac{2}{3} \quad (6)$$

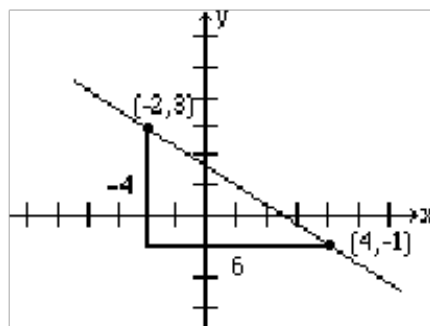


Figure 6

To give the reader a better understanding, both the vertical change, -4 , and the horizontal change, 6 , are shown in the above figure.

When two points are given, it does not matter which point is denoted as (x_1, y_1) and which (x_2, y_2) . The value for the slope will be the same. For example, if we choose $(x_1, y_1) = (4, -1)$ and $(x_2, y_2) = (-2, 3)$, we will get the same value for the slope as we obtained earlier. The steps involved are as follows.

$$m = \frac{3 - (-1)}{-2 - 4} = \frac{4}{-6} = -\frac{2}{3} \quad (7)$$

The student should further observe that if a line rises when going from left to right, then it has a positive slope; and if it falls going from left to right, it has a negative slope.

Example 7

Find the slope of the line that passes through the points $(2, 3)$ and $(2, -1)$, and graph.

Solution

Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (2, -1)$ then the slope

$$m = \frac{-1 - 3}{2 - 2} = -\frac{4}{0} = \text{undefined} \quad (8)$$

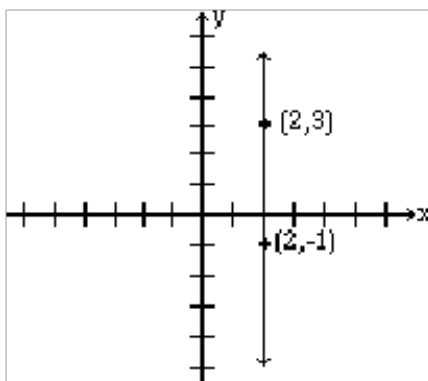


Figure 7

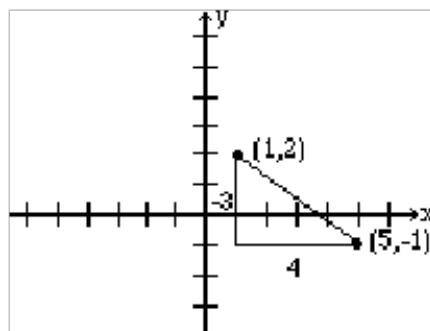
NOTE: The slope of a vertical line is undefined.

Example 8

Graph the line that passes through the point $(1, 2)$ and has slope $-\frac{3}{4}$.

Solution

Slope equals $\frac{\text{rise}}{\text{run}}$. The fact that the slope is $-\frac{3}{4}$, means that for every rise of -3 units (fall of 3 units) there is a run of 4. So if from the given point $(1, 2)$ we go down 3 units and go right 4 units, we reach the point $(5, -1)$. The following graph is obtained by connecting these two points.

**Figure 8**

Alternatively, since $\frac{3}{-4}$ represents the same number, the line can be drawn by starting at the point (1, 2) and choosing a rise of 3 units followed by a run of -4 units. So from the point (1, 2), we go up 3 units, and to the left 4, thus reaching the point $(-3, 5)$ which is also on the same line. See figure below.

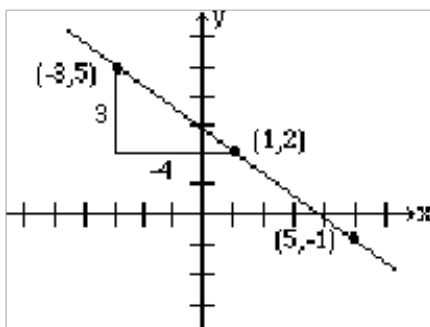


Figure 9

Example 9

Find the slope of the line $2x + 3y = 6$.

Solution

In order to find the slope of this line, we will choose any two points on this line.

Again, the selection of x and y intercepts seems to be a good choice. The x -intercept is $(3, 0)$, and the y -intercept is $(0, 2)$. Therefore, the slope is

$$m = \frac{2 - 0}{0 - 3} = -\frac{2}{3}. \quad (9)$$

The graph below shows the line and the intercepts: x and y .

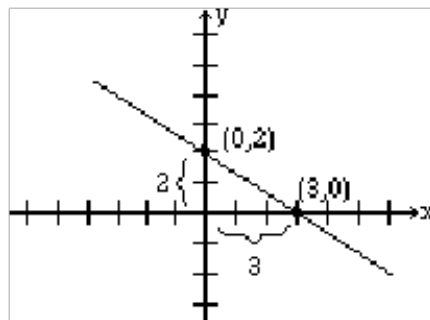


Figure 10

Example 10

Find the slope of the line $y = 3x + 2$.

Solution

We again find two points on the line. Say (0, 2) and (1, 5).

Therefore, the slope is $m = \frac{5-2}{1-0} = \frac{3}{1} = 3$.

Look at the slopes and the y-intercepts of the following lines.

The line	Slope	y-intercept
$y = 3x + 2$	3	2
$y = -2x + 5$	-2	5
$y = 3/2x - 4$	3/2	-4

Table 4

It is no coincidence that when an equation of the line is solved for y , the coefficient of the x term represents the slope, and the constant term represents the y-intercept.

In other words, for the line $y = mx + b$, m is the slope, and b is the y-intercept.

Example 11

Determine the slope and y-intercept of the line $2x + 3y = 6$.

Solution

We solve for y .

$$2x + 3y = 6 \quad (10)$$

$$3y = -2x + 6 \quad (11)$$

$$y = -2/3x + 2 \quad (12)$$

The slope = the coefficient of the x term = $-2/3$

The y-intercept = the constant term = 2.

4 Determining the Equation of a Line

4.1 Section Overview

In this section, you will learn to:

1. Find an equation of a line if a point and the slope are given.
2. Find an equation of a line if two points are given.

So far, we were given an equation of a line and were asked to give information about it. For example, we were asked to find points on it, find its slope and even find intercepts. Now we are going to reverse the process. That is, we will be given either two points, or a point and the slope of a line, and we will be asked to find its equation.

An equation of a line can be written in two forms, the **slope-intercept form** or the **standard form**.

The Slope-Intercept Form of a Line: $y = mx + b$

A line is completely determined by two points, or a point and slope. So it makes sense to ask to find the equation of a line if one of these two situations is given.

Example 12

Find an equation of a line whose slope is 5, and y-intercept is 3.

Solution

In the last section we learned that the equation of a line whose slope = m and y-intercept = b is $y = mx + b$.

Since $m = 5$, and $b = 3$, the equation is $y = 5x + 3$.

Example 13

Find the equation of the line that passes through the point (2, 7) and has slope 3.

Solution

Since $m = 3$, the partial equation is $y = 3x + b$.

Now b can be determined by substituting the point (2, 7) in the equation $y = 3x + b$.

$$7 = 3(2) + b \quad (13)$$

$$b = 1 \quad (14)$$

Therefore, the equation is $y = 3x + 1$.

Example 14

Find an equation of the line that passes through the points $(-1, 2)$, and $(1, 8)$.

Solution

$$m = \frac{8 - 2}{1 - (-1)} = \frac{6}{2} = 3 \quad (15)$$

So the partial equation is $y = 3x + b$

Now we can use either of the two points $(-1, 2)$ or $(1, 8)$, to determine b .

Substituting $(-1, 2)$ gives

$$2 = 3(-1) + b \quad (16)$$

$$5 = b \quad (17)$$

So the equation is $y = 3x + 5$.

Example 15

Find an equation of the line that has x-intercept 3, and y-intercept 4.

Solution

x-intercept = 3, and y-intercept = 4 correspond to the points $(3, 0)$, and $(0, 4)$, respectively.

$$m = \frac{4 - 0}{0 - 3} = \frac{4}{-3} \quad (18)$$

So the partial equation for the line is $y = -4/3x + b$

Substituting $(0, 4)$ gives

$$4 = -4/3(0) + b \quad (19)$$

$$4 = b \quad (20)$$

Therefore, the equation is $y = -4/3x + 4$.

The Standard form of a Line: $Ax + By = C$

Another useful form of the equation of a line is the Standard form.

Let L be a line with slope m , and containing a point (x_1, y_1) . If (x, y) is any other point on the line L , then by the definition of a slope, we get

$$m = \frac{y - y_1}{x - x_1} \quad (21)$$

$$y - y_1 = m(x - x_1) \quad (22)$$

The last result is referred to as the **point-slope form** or point-slope formula. If we simplify this formula, we get the equation of the line in the standard form, $Ax + By = C$.

Example 16

Using the point-slope formula, find the standard form of an equation of the line that passes through the point $(2, 3)$ and has slope $-3/5$.

Solution

Substituting the point $(2, 3)$ and $m = -3/5$ in the point-slope formula, we get

$$y - 3 = -3/5(x - 2) \quad (23)$$

Multiplying both sides by 5 gives us

$$5(y - 3) = -3/5(x - 2) \quad (24)$$

$$5y - 15 = -3x + 6 \quad (25)$$

$$3x + 5y = 21 \quad (26)$$

Example 17

Find the standard form of the line that passes through the points $(1, -2)$, and $(4, 0)$.

Solution

$$m = \frac{0 - (-2)}{4 - 1} = \frac{2}{3} \quad (27)$$

The point-slope form is

$$y - (-2) = 2/3(x - 1) \quad (28)$$

Multiplying both sides by 3 gives us

$$3(y + 2) = 2(x - 1) \quad (29)$$

$$3y + 6 = 2x - 2 \quad (30)$$

$$-2x + 3y = -8 \quad (31)$$

$$2x - 3y = 8 \quad (32)$$

We should always be able to convert from one form of an equation to another. That is, if we are given a line in the slope-intercept form, we should be able to express it in the standard form, and vice versa.

Example 18

Write the equation $y = -2/3x + 3$ in the standard form.

Solution

Multiplying both sides of the equation by 3, we get

$$3y = -2x + 9 \quad (33)$$

$$2x + 3y = 9 \quad (34)$$

Example 19

Write the equation $3x - 4y = 10$ in the slope-intercept form.

Solution

Solving for y , we get

$$-4y = -3x + 10 \quad (35)$$

$$y = 3/4x - 5/2 \quad (36)$$

Finally, we learn a very quick and easy way to write an equation of a line in the standard form. But first we must learn to find the slope of a line in the standard form by inspection.

By solving for y , it can easily be shown that the slope of the line $Ax + By = C$ is $-A/B$. The reader should verify.

Example 20

Find the slope of the following lines, by inspection.

- a. $3x - 5y = 10$
- b. $2x + 7y = 20$
- c. $4x - 3y = 8$

Solution

- a. $A = 3$, $B = -5$, therefore, $m = -\frac{3}{-5} = \frac{3}{5}$
- b. $A = 2$, $B = 7$, therefore, $m = -\frac{2}{7}$
- c. $m = -\frac{4}{-3} = \frac{4}{3}$

Now that we know how to find the slope of a line in the standard form by inspection, our job in finding the equation of a line is going to be very easy.

Example 21

Find an equation of the line that passes through $(2, 3)$ and has slope $-4/5$.

Solution

Since the slope of the line is $-4/5$, we know that the left side of the equation is $4x + 5y$, and the partial equation is going to be

$$4x + 5y = c \quad (37)$$

Of course, c can easily be found by substituting for x and y .

$$4(2) + 5(3) = c \quad (38)$$

$$23 = c \quad (39)$$

The desired equation is

$$4x + 5y = 23. \quad (40)$$

If you use this method often enough, you can do these problems very quickly.

5 Applications

Now that we have learned to determine equations of lines, we get to apply these ideas in real-life equations.

Example 22

A taxi service charges \$0.50 per mile plus a \$5 flat fee. What will be the cost of traveling 20 miles? What will be cost of traveling x miles?

Solution

$$\text{The cost of traveling 20 miles} = y = (.50)(20) + 5 = 10 + 5 = 15 \quad (41)$$

$$\text{The cost of traveling } x \text{ miles} = y = (.50)(x) + 5 = .50x + 5 \quad (42)$$

In this problem, \$0.50 per mile is referred to as the **variable cost**, and the flat charge \$5 as the **fixed cost**. Now if we look at our cost equation $y = .50x + 5$, we can see that the variable cost corresponds to the slope and the fixed cost to the y-intercept.

Example 23

The variable cost to manufacture a product is \$10 and the fixed cost \$2500. If x represents the number of items manufactured and y the total cost, write the cost function.

Solution

The fact that the variable cost represents the slope and the fixed cost represents the y-intercept, makes $m = 10$ and $y = 2500$.

Therefore, the cost equation is $y = 10x + 2500$.

Example 24

It costs \$750 to manufacture 25 items, and \$1000 to manufacture 50 items. Assuming a linear relationship holds, find the cost equation, and use this function to predict the cost of 100 items.

Solution

We let x = the number of items manufactured, and let y = the cost.

Solving this problem is equivalent to finding an equation of a line that passes through the points (25, 750) and (50, 1000).

$$m = \frac{1000-750}{50-25} = 10$$

Therefore, the partial equation is $y = 10x + b$

By substituting one of the points in the equation, we get $b = 500$

Therefore, the cost equation is $y = 10x + 500$

Now to find the cost of 100 items, we substitute $x = 100$ in the equation $y = 10x + 500$

So the cost = $y = 10(100) + 500 = 1500$

Example 25

The freezing temperature of water in Celsius is 0 degrees and in Fahrenheit 32 degrees. And the boiling temperatures of water in Celsius, and Fahrenheit are 100 degrees, and 212 degrees, respectively. Write a conversion equation from Celsius to Fahrenheit and use this equation to convert 30 degrees Celsius into Fahrenheit.

Solution

Let us look at what is given.

Centigrade	Fahrenheit
0	32
100	212

Table 5

Again, solving this problem is equivalent to finding an equation of a line that passes through the points (0, 32) and (100, 212).

Since we are finding a linear relationship, we are looking for an equation $y = mx + b$, or in this case $F = mC + b$, where x or C represent the temperature in Celsius, and y or F the temperature in Fahrenheit.

$$\text{slope } m = \frac{212 - 32}{100 - 0} = \frac{9}{5} \quad (43)$$

The equation is $F = \frac{9}{5}C + b$

Substituting the point (0, 32), we get

$$F = \frac{9}{5}C + 32. \quad (44)$$

Now to convert 30 degrees Celsius into Fahrenheit, we substitute $C = 30$ in the equation

$$F = \frac{9}{5}C + 32 \quad (45)$$

$$F = \frac{9}{5}(30) + 32 = 86 \quad (46)$$

Example 26

The population of Canada in the year 1970 was 18 million, and in 1986 it was 26 million. Assuming the population growth is linear, and x represents the year and y the population, write the function that gives a relationship between the time and the population. Use this equation to predict the population of Canada in 2010.

Solution

The problem can be made easier by using 1970 as the base year, that is, we choose the year 1970 as the year zero. This will mean that the year 1986 will correspond to year 16, and the year 2010 as the year 40.

Now we look at the information we have.

Solving this problem is equivalent to finding an equation of a line that passes through the points $(0, 18)$ and $(16, 26)$.

$$m = \frac{26 - 18}{16 - 0} = \frac{1}{2} \quad (47)$$

The equation is $y = \frac{1}{2}x + b$

Substituting the point $(0, 18)$, we get

$$y = \frac{1}{2}x + 18 \quad (48)$$

Now to find the population in the year 2010, we let $x = 40$ in the equation

$$y = \frac{1}{2}x + 18 \quad (49)$$

$$y = \frac{1}{2}(40) + 18 = 38 \quad (50)$$

So the population of Canada in the year 2010 will be 38 million.

Year	Population
0 (1970)	18 million
16 (1986)	26 million

Table 6

6 More Applications

6.1 Section Overview

In this section, you will learn to:

1. Solve a linear system in two variables.
2. Find the equilibrium point when a demand and a supply equation are given.
3. Find the break-even point when the revenue and the cost functions are given.

In this section, we will do application problems that involve the intersection of lines. Therefore, before we proceed any further, we will first learn how to find the intersection of two lines.

Example 27

Find the intersection of the line $y = 3x - 1$ and the line $y = -x + 7$.

Solution

We graph both lines on the same axes, as shown below, and read the solution $(2, 5)$.

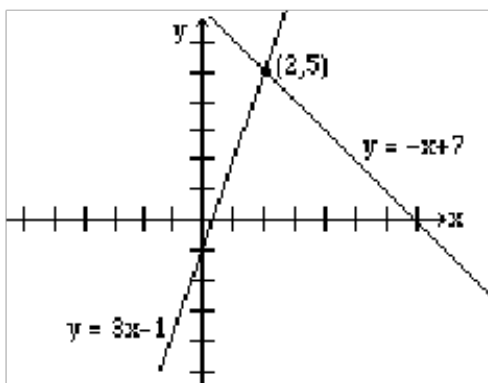


Figure 11

Finding an intersection of two lines graphically is not always easy or practical; therefore, we will now learn to solve these problems algebraically.

At the point where two lines intersect, the x and y values for both lines are the same. So in order to find the intersection, we either let the x -values or the y -values equal.

If we were to solve the above example algebraically, it will be easier to let the y -values equal. Since $y = 3x - 1$ for the first line, and $y = -x + 7$ for the second line, by letting the y -values equal,

we get

$$\begin{aligned} 3x - 1 &= -x + 7 \\ 4x &= 8 \\ x &= 2 \end{aligned} \tag{51}$$

By substituting $x = 2$ in any of the two equations, we obtain $y = 5$.

Hence, the solution is $(2, 5)$.

One common algebraic method used in solving systems of equations is called the **elimination method**. The object of this method is to eliminate one of the two variables by adding the left and right sides of the equations together. Once one variable is eliminated, we get an equation that has only one variable for which it can be solved. Finally, by substituting the value of the variable that has been found in one of the original equations, we get the value of the other variable. The method is demonstrated in the example below.

Example 28

Find the intersection of the lines $2x + y = 7$ and $3x - y = 3$ by the elimination method.

Solution

We add the left and right sides of the two equations.

$$\begin{aligned} 2x + y &= 7 \\ \underline{3x - y} &= \underline{3} \\ 5x &= 10 \end{aligned} \tag{52}$$

$$x = 2 \tag{53}$$

Now we substitute $x = 2$ in any of the two equations and solve for y .

$$2(2) + y = 7 \tag{54}$$

$$y = 3 \tag{55}$$

Therefore, the solution is $(2, 3)$.

Example 29

Solve the system of equations $x + 2y = 3$ and $2x + 3y = 4$ by the elimination method.

Solution

If we add the two equations, none of the variables are eliminated. But the variable x can be eliminated by multiplying the first equation by -2 , and leaving the second equation unchanged.

$$\begin{aligned} -2x - 4y &= -6 \\ \underline{2x + 3y} &= \underline{4} \\ -y &= -2 \end{aligned} \tag{56}$$

$$y = 2 \tag{57}$$

Substituting $y = 2$ in $x + 2y = 3$, we get

$$x + 2(2) = 3 \quad (58)$$

$$x = -1 \quad (59)$$

Therefore, the solution is $(-1, 2)$.

Example 30

Solve the system of equations $3x - 4y = 5$ and $4x - 5y = 6$.

Solution

This time, we multiply the first equation by -4 and the second by 3 before adding. (The choice of numbers is not unique.)

$$\begin{aligned} -12x + 16y &= -20 \\ 12x - 15y &= 18 \\ \hline y &= -2 \end{aligned} \quad (60)$$

By substituting $y = -2$ in any one of the equations, we get $x = -1$. Hence the solution $(-1, -2)$.

Supply, Demand and the Equilibrium Market Price

In a free market economy the supply curve for a commodity is the number of items of a product that can be made available at different prices, and the demand curve is the number of items the consumer will buy at different prices. As the price of a product increases, its demand decreases and supply increases. On the other hand, as the price decreases the demand increases and supply decreases. The **equilibrium price** is reached when the demand equals the supply.

Example 31

The supply curve for a product is $y = 1.5x + 10$ and the demand curve for the same product is $y = -2.5x + 34$, where x is the price and y the number of items produced. Find the following.

- How many items will be supplied at a price of \$10?
- How many items will be demanded at a price of \$10?
- Determine the equilibrium price.
- How many items will be produced at the equilibrium price?

Solution

- We substitute $x = 10$ in the supply equation, $y = 1.5x + 10$, and the answer is $y = 25$.
- We substitute $x = 10$ in the demand equation, $y = -2.5x + 34$, and the answer is $y = 9$.
- By letting the supply equal the demand, we get

$$1.5x + 10 = -2.5x + 34 \quad (61)$$

$$4x = 24 \quad (62)$$

$$x = 6 \quad (63)$$

- d. We substitute $x = 6$ in either the supply or the demand equation and we get $y = 19$. The graph below shows the intersection of the supply and the demand functions and their point of intersection, $(6, 19)$.

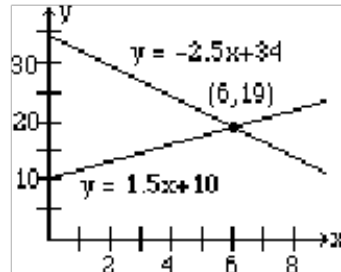


Figure 12

Break-Even Point

In a business, the profit is generated by selling products. If a company sells x number of items at a price P , then the revenue R is P times x , i.e., $R = P \cdot x$. The production costs are the sum of the variable costs and the fixed costs, and are often written as $C = mx + b$, where x is the number of items manufactured.

A company makes a profit if the revenue is greater than the cost, and it shows a loss if the cost is greater than the revenue. The point on the graph where the revenue equals the cost is called the **Break-even point**.

Example 32

If the revenue function of a product is $R = 5x$ and the cost function is $y = 3x + 12$, find the following.

- If 4 items are produced, what will the revenue be?
- What is the cost of producing 4 items?
- How many items should be produced to break-even?
- What will be the revenue and the cost at the break-even point?

Solution

- We substitute $x = 4$ in the revenue equation $R = 5x$, and the answer is $R = 20$.
- We substitute $x = 4$ in the cost equation $C = 3x + 12$, and the answer is $C = 24$.

c. By letting the revenue equal the cost, we get

$$5x = 3x + 12 \quad (64)$$

$$x = 6 \quad (65)$$

d. We substitute $x = 6$ in either the revenue or the cost equation, and we get $R = C = 30$.
The graph below shows the intersection of the revenue and the cost functions and their point of intersection, $(6, 30)$.

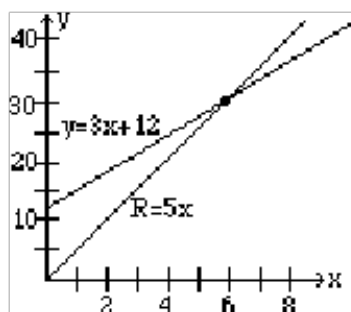


Figure 13