Problem

Solving and the English Language Learner

Various techniques can enhance mathematics instruction for English language learners.

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Min, an English language learner (ELL), came to the United States from Vietnam and enrolled in sixth grade. After being in the country for sixth months, he now has a simple command of English and can carry on short conversations with his teachers and peers. However, mathematics class has been very difficult for him, because he understands little of his teacher's explanation. Furthermore, he feels helpless whenever he is asked to solve word problems, although he does not have any difficulties with computation. Min wishes that someone would explain the mathematics to him in his native language so that he could understand and solve word problems.

This is a typical story of an ELL who struggles with English, especially in the beginning stage of language acquisition. Min's story shows the plight of many ELLs who do not understand the language of mathematics because of language barriers. The number of ELLs is growing rapidly. From 1991 to 2001, the ELL enrollment in public schools in the United States increased by 95 percent, whereas the general student population increased only 12 percent (Padolsky 2002). This influx into public schools by students whose native language is not English is of great concern to educators. ELLs' academic achievement, especially in mathematics, is important, because

mathematics has been critically linked to earning power and job prospects. Minorities are less likely to choose college majors in technology-related or science fields that require mathematics as a core prerequisite. They are also underrepresented in those fields.

For example, comparisons among Latinos, African Americans, and Caucasians in the level of income, occupation, and basic education show that 65 percent of Latinos with limited English proficiency, 50 percent of Latinos, 44 percent of African Americans, and 29 percent of Caucasians made less than $30,000 a year in 2002–2003 (AFT 2004). The National Assessment of Educational Progress, published in 2003, indicated that only 11 percent of Latinos scored at or above grade level (AFT 2004). A large percentage of Latino students tend to drop out before the tenth grade. In 2003, 33.7 percent of immigrant Latinos ages 16 through 19 dropped out, whereas for Caucasians, that number is only 8.2 percent (AFT 2004; Fry 2003).

Statistics showing the increase in ELL populations and a persistent achievement gap between these populations and Caucasians who are proficient in English speak volumes to mathematics educators. It enforces the message that we must serve ELLs effectively to reverse this trend. In addition, with new forms of accountability, educators can no longer assume that ELLs need to master English first before they learn subject matter. Considering NCTM's Equity Principle (NCTM 2000), there is an urgent need for mathematics teachers to find ways to accommodate ELLs so that they not only become successful in mathematics but also see positive gains in their earnings and occupational choices.

IS MATHEMATICS A UNIVERSAL LANGUAGE?

Many educators assume that mathematics is a universal language that cuts across all localities. It is generally assumed that mathematics is the one content area in which ELLs can perform competitively and that low English proficiency is not necessarily a disadvantage because mathematics is not about language but about numbers (Brown 2005). Because mathematics uses symbols and algorithms that are not specific to one's way of speaking, mathematics is assumed to be a universal language. A subtraction problem is a subtraction problem whether it is taught in English in the United States, in Spanish in Mexico, or in Japanese in Japan. This notion of the universality of mathematics is, however, a fallacy.

Although symbols and numbers are standard across many countries, there are subtle differences in the way in which they are used or expressed. This subtlety is large enough to cause confusion for ELLs when they are learning mathematics. **Table 1** shows examples that initially appear to be insignificant but that can cause confusion for ELLs. Furthermore, differences are not limited to the way that the numbers are recorded. The way that numbers are *read* is also different, which can also cause confusion. In English, the numeral 10,000, is read as "ten thousand," and 100,000 is read as "one hundred thousand." In Korea, however, a different word, "man," indicates 10,000, and 100,000 is read as "ten man." Therefore, it is not uncommon for a Korean ELL to make a mental note of the quantity 1,000,000 when they hear "one hundred thousand." Not being able to correctly represent the numbers they hear would greatly affect the processing of mathematics. This illustrates one of the challenges that ELLs face in an English-only mathematics classroom.

IS MATHEMATICS CULTURALLY NEUTRAL?

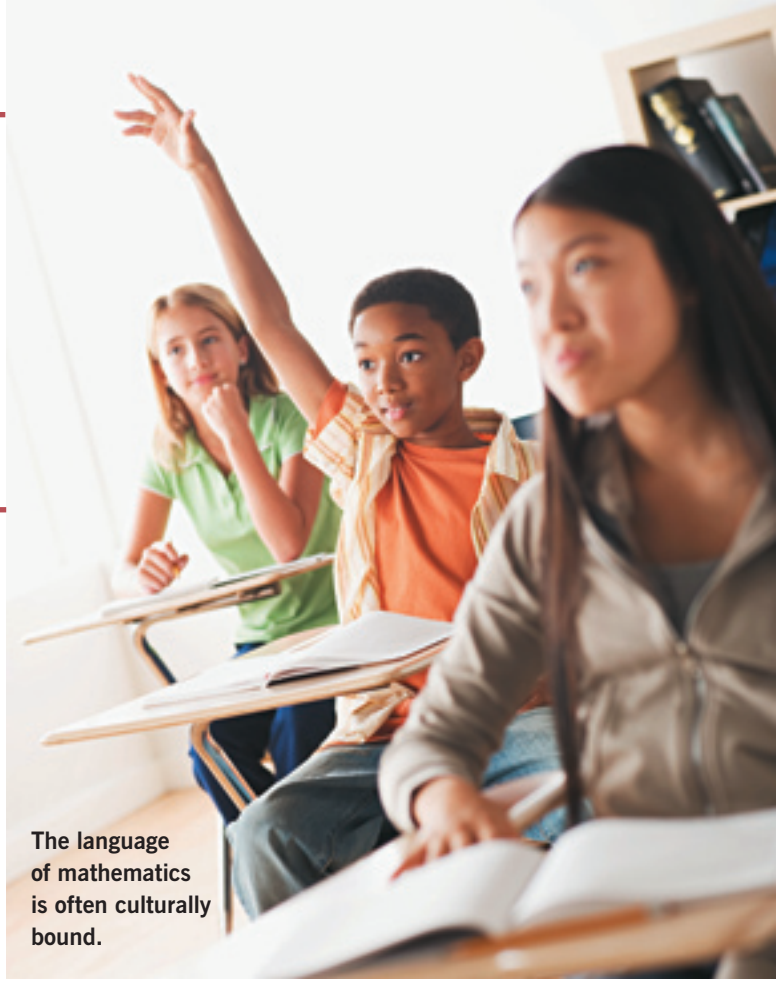
Another assumption often made by educators is that mathematics is a culturally neutral subject because it involves numbers. In other words, regardless of the students' cultural or prior knowledge, students of all backgrounds can solve problems when they have adequate mathematics skills and knowledge. For instance, NCTM's Standards recommend the use of real-world situations to provide contextual clues to aid in problem solving. However, if the context used in word problems is unfamiliar to ELLs, it actually poses more problems than help during the problem-solving process. The following word problem, focusing on estimation skills, was given to a group of seventh-grade students:

> Example 1: Noelle is hanging streamers for a party and needs to attach streamers from the corners of the room to the center of the room.

Table 1 Systems that can cause confusion among ELLs

Examples	U.S. Notation	Foreign Country Notation
Dates	5/26/50 Month/day/year	26/5/50 (i.e., Mexico) Day/month/year
Times	9:35 a.m. 3:45 p.m. 12-hour clock	09:35 15:45 24-hour clock (i.e., France)
Place-value notations, or marking place value	9,427,813	9.427.813
Marking decimal places	15.6 (fifteen and six-tenths)	15,6 (fifteen and six-tenths)

Having predictable routines and signals, among the easiest strategies to implement, are extremely important in reducing stress for ELLs



The language of mathematics is often culturally bound.

> If the room is 34 feet by 34 feet, what will you need to estimate to find the total number of the streamers used to decorate the room?

The context of decorating the room with streamers would not be an issue to English-speaking students. However, Maria, an ELL, asked for a clarification of the word *streamers* because she had just learned the word *stream*, referring to a body of water, from reading a story in her ESL class. She understood that the situation in the question did not involve a stream outdoors but a room indoors. All she could think about was the stream of water; she did not know what to make of the word *streamers* that was associated with the room.

This confusion by Maria highlights how the language in a mathematics question can be just as culturally bound as in any other subject. This problem asks students to apply estimation in a culturally bound context. If ELLs do not know that streamers are colored paper strips, they will not recognize that this question involves estimation. This contextually based word problem inadvertently becomes less about mathematics and more about vocabulary.

STRATEGIES TO SUPPORT ENGLISH LANGUAGE LEARNERS

Research has shown that ELLs perform comparably to their English-speaking counterparts in computational mathematics but not in word problems (Brown 2005). The fact that ELLs lag behind fluent English speakers in problem solving shows that mathematics performance may be compounded by English language and cultural knowledge. Since the language of mathematics is not universal and mathematics contexts are culturally bound, mathematics teachers must be aware of their role as a language teacher when working with ELLs.

A belief held by many classroom teachers is that accommodating ELLs is an extra step that is otherwise unnecessary. However, instructional strategies that accommodate these students in a mathematics class will benefit everyone. In other words, general education students who perform below grade level would benefit from the instructional strategies that target ELLs (Brown 2007). For example, a word problem might read,

> Find a number that decreased by 30 is 4 times its opposite.

To help ELLs comprehend this word problem, a teacher needs to take time to explain the vocabulary and break down the sentence into comprehensible chunks of phrases and words. At the same time, this type of scaffolding would definitely help many English-speaking students, as well. **Figure 1** models a think-aloud protocol, showing a sequence and a teacher's verbal explanation. After helping students understand what this problem is asking, the teacher can model how to solve the equation.

As the example indicates, the general recommendations applied to ELLs could be equally applied to all students' mathematical learning. The Principles and Process Standards from *Principles and Standards for School Mathematics* (NCTM 2000), used with ELLs in mind, reveal a connection between the two groups. The remainder of this article outlines additional strategies that are beneficial to ELLs and, thus, all students.

Create Low-Anxiety Environments

The Teaching Principle states, "Effective teaching requires a challenging and supportive learning environment" (NCTM 2000, p. 18). For language acquisition and learning to occur, students must feel safe and comfortable

Figure 1 Modeling a think-aloud protocol, showing a sequence and a teacher's verbal explanation, which answers "Find a number that decreased by 30 is 4 times its opposite"

Teacher's Discourse	Steps in the Think-Aloud Protocol
OK, we don't know what this number is. This number is decreased by 30. What does *decreased* mean? It means *reduced*, *cut*, *dropped*, or *shrunk*.	Substitute potentially difficult words with easier ones.
A number decreased by 30 means that this number was reduced, cut, dropped, or shrunk by 30.	Paraphrase the sentence with easier vocabulary.
At the same time, this number we need to find is "4 times of the opposite number."	Unpack the problem that requires multiple steps.
First of all, what does *opposite* mean? High low, big small, heavy light. What then is an opposite number? It could be 5 and -5, 2 and -2, 10 and -10. These numbers are on the opposite side on the number line.	Explain abstract words with concrete examples.
Now that I figured out what this problem is about, I want to put the word problem into my own words, because that helps me understand the problem better. I need to find this number. This number is dropped by 30. This same number is 4 times the opposite of this number.	Put the question back together in simpler terms.
Now, I am going to show you how I change my own words into an equation: • I would call this number an n. • This number is dropped by 30. I have to subtract 30 from n, which would be $n - 30$. • The opposite of this number would be as in 2 and -2. Then it would be n and $-n$. • Four times the opposite number would be $4(-n)$. • The equation is then $n - 30 = 4(-n)$.	Discuss how to make this problem into an equation.

in their environment (Krashen 2003). ELLs who are learning a new language will inadvertently make many mistakes. Peer approval, especially among adolescents, is important. Thus, it is critical that students learn in an environment in which they know that their attempts to participate will be accepted and encouraged (Rothenberg and Fisher 2007).

However, in creating low-anxiety environments, teachers should not relax their expectations for ELLs. High expectations, although important for all students, play a profound role in the success of ELLs in particular (Gutierrez 2002; Slavin and Calderon 2001). Lower expectations for ELLs is often manifested in subtle ways and allow these students to become passive learners. Ways to reduce anxiety in the mathematics classroom while keeping expectations high are discussed below.

Allow ELLs to Solve Problems in Familiar Ways

ELLs know mathematics and can find solutions to complex problems. However, they might not follow the steps prescribed in the textbook. It is important to allow students to solve problems in ways that are familiar to them, as long as the approaches are mathematically sound and appropriate. This strategy is recommended for use in the mathematics classroom with any student, but it is particularly meaningful for ELLs. In many cases, they have learned an algorithm that is viewed as nontraditional in the United States. As a result, ELLs may have trouble shifting to the algorithm accepted as being the most efficient by U.S. teachers.

Allow Diverse Problem-Solving Techniques

ELLs bring with them a variety of problem-solving strategies. When the teacher identifies an ELL's alternative strategy, this becomes an opportunity for rich mathematical conversation about problem solving. The approach of making multiple strategies an overt topic of attention in class throughout the school year would benefit both ELLs and English-speaking students. First of all, it teaches all students the importance of the mathematical processes of problem solving. It helps them see that more than one way is possible to solve a problem and realize that mathematics is more than applying a set of rules. Second, this inclusive teaching practice provides equal footing to ELLs because they are being treated the same as the other students. It also helps them realize that they are not seen as deficient because they solve problems differently.

Establish Predictable Routines and Signals

Having predictable routines and signals are among the easiest strategies to implement, yet they are extremely important in reducing stress for ELLs. When they know the sequence of events and instructional activities, they are able to focus more energy on

instruction (Herrell and Jordan 2007). Routines include a designated spot where homework is posted; places for submitting assignments; and areas designated for group work, partner work, or individual work. Signals such as using hands or flashing the lights also give ELLs a sense of order in their learning and minimize confusion about what is going on. It is important for teachers to clearly communicate, for example, when one activity is finished and another is beginning. These routines should be modeled for students, ELLs in particular.

Use Visual Cues

Visual cues are important for aiding understanding in vocabulary development. Write key words on the board or overhead projector as you say them, because ELLs may not recognize the spoken words as being those in written form. This idea can be extended to include graphic organizers to help ELLs see relationships between words or concepts using such items as pictures, props, and photographs.

Contextualize class directions, since it is challenging for ELLs to follow verbal directions. For example, "Take out your book" should be followed by holding up a mathematics book. If the teacher explains that there are three steps to follow, he or she should hold up three fingers and count off a finger for each step. The concurrent verbal explanation and physical demonstration is one of the simplest yet most powerful strategies to use with ELLs (Herrell and Jordan 2007).

In the case of the streamer problem discussed earlier, having streamers on hand and decorating a bulletin board can solve the language problem. If the issue involves the mathematical language, however, the key is to revert to using manipulatives and other hands-on materials. It also helps to translate among representations (concrete, pictorial, and symbolic) and the spoken and written language. Using these different kinds of visual cues will help ELLs build strong connections to the language based on their past experiences or a current lesson.

Technical mathematical vocabulary, dense word problems, a lack of cultural knowledge assumed by teachers, and culturally different ways of using numbers cause ELLs to struggle

Assign Student Buddies

Partner work is particularly effective with ELLs, because it provides opportunities for verbal interaction and support (Echevarria, Vogt, and Short 2000; Herrell and Jordan 2007). Therefore, pair ELLs with student buddies to help them learn the routine and provide assistance with routines. They will appreciate the help and support of a peer, and the volunteer will feel satisfaction in helping a fellow student. It is useful to give the volunteer some guidelines and help in how to pose questions that support thinking, when to translate into the first language, and how to encourage vocabulary development. Although it may seem beneficial to pair ELLs with someone who also speaks their language, it can be better not to do so, because this may lead to a reliance on the buddy as a translator, placing an undue burden on that person (Reiss 2007).

Become a Language Teacher

As discussed so far, mathematics learning is as much language-based as any other content areas. It is important, therefore, that mathematics teachers help ELLs acquire English through learning the language of mathematics. Word problems in particular are more challenging to ELLs, not from their lack of computation skills but because of their limited English proficiency. Mathematics vocabulary (e.g., *least common denominator* and *greatest common factor*) is highly technical and abstract in nature. These terms are rarely heard in daily life, which make them more difficult for ELLs to acquire.

The think-aloud example involving algebra in **figure 1** attests to the need for mathematics teachers to take on the role of language teacher, as well. Conceptual understanding, which is essential in mathematics, cannot occur if the teacher does not reduce the language barrier for ELLs. Teachers can help ELLs gain access to mathematical concepts by substituting difficult vocabulary for more commonly used words, paraphrasing unfamiliar expressions, or unpacking complex statements or questions into several simple sentences. Comprehending the language of mathematics should positively impact ELLs' performance when solving word problems.

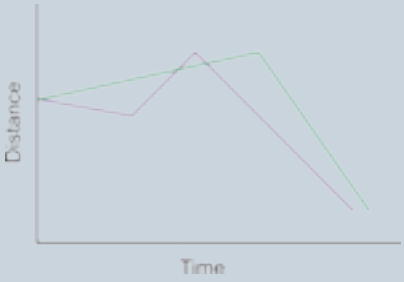
Use Advanced Organizers as Mathematical Connections

Students will benefit when lessons link their past experiences to new concepts. Organizers help ELLs activate their prior knowledge and, in so doing, help make connections to new information. These organizers can take on different forms, depending on

Fig. 2 Students explain what the graph is showing.

What's Your Story?

The graph below shows the distance of two objects from a fixed point during a specified time limit.



A. Write a story to match the graph. Make sure that is it is clear which objects or characters correspond to which graph and what the scale is on each axis.
B. Explain your story, and how it is related to the features of the graph.

students' familiarity with the concepts of the lesson and the developmental level of the students. Expository organizers link new information with prior knowledge when the material is relatively unfamiliar to the student. Simply asking students to explain the relationship between two mathematical terms can encourage them to make connections that they had not previously made, thereby strengthening their knowledge and facilitating recall. Comparative organizers integrate new material with similar material already understood.

PUTTING IT ALL TOGETHER: AN EXAMPLE

The problem in **figure 2** asks students to create a story that could represent the data in the graph. An expository organizer would assume that students had not seen graphs similar to this one before. Writing prompts might include the following:

- Where are the objects closest to the target point?
- Where are they farthest from the target point?
- In the context of the graph, what does the horizontal line indicate?
- What does a downward sloping line indicate?
- What does an upward sloping line indicate?

A comparative organizer would assume that students had some familiarity with this type of graph, which is comparing distance and time. Perhaps students had experimented with calculator-based rangers (CBRs) to collect and analyze motion data. The teacher would then begin a discussion that would ask students to relate this graph to one they had seen as they documented their motion. As ELLs acquire more English, advanced organizers can be become more abstract and the use of Venn diagrams, oral readings, and classroom debates become useful. However, ELLs will continue to need concrete props to demonstrate new vocabulary words (Herrell and Jordan 2007).

The student work in **figures 3a** and **3b** are not offered as exemplars but as examples of students using their prior knowledge to approach the tasks. In **figure 3a**, one student employed a cultural phenomenon, a cartoon character, to understand and explain the features of the graph. Although the explanation in **figure 3b** seems more universal, the idea of "roller-skating at park" may need to be explained. Both of these examples, however, use everyday language to explain features of the graph. The teacher engaged the students in a discussion about mathematical correctness, and the students began to see a need to use standard mathematical language. At this point in the discussion, students began to augment context-specific terms and phrases such as "drove" and "slammed on the brakes" with common mathematical terms such as "increased the distance" and "changed direction." Through this discussion, the teacher was able to support the needs of ELLs. At the same time, she was enhancing the needs of all students by having them establish the connections between the graphical representations, the story in common English, and the technical description in mathematical terms.

CONCLUSION

ELLs' aptitude in computational mathematics is comparable to their English-speaking counterparts, yet they lag behind in their ability to solve word problems. The technical nature of mathematical vocabulary, densely expressed ideas in word problems, a lack of cultural knowledge assumed by teachers, and culturally different ways of expressing and using numbers cause ELLs to struggle. If we are to help ELLs, such as Min, to understand mathematical concepts, teachers should explore how mathematics is language-bound and how language acquisition is the biggest obstacle to learning mathematics for ELLs. Thus, one of the most important things that mathematics teachers can do is to be aware of their role as language teachers (Brown 2007). Using the strategies suggested in this article, such as scaffolding instruction and accommodating their linguistic needs, mathematics teachers can help ELLs both succeed and increase their mathematics achievement. Unless ELLs experience some strategies that help them with language acquisition, they will most certainly be left behind.

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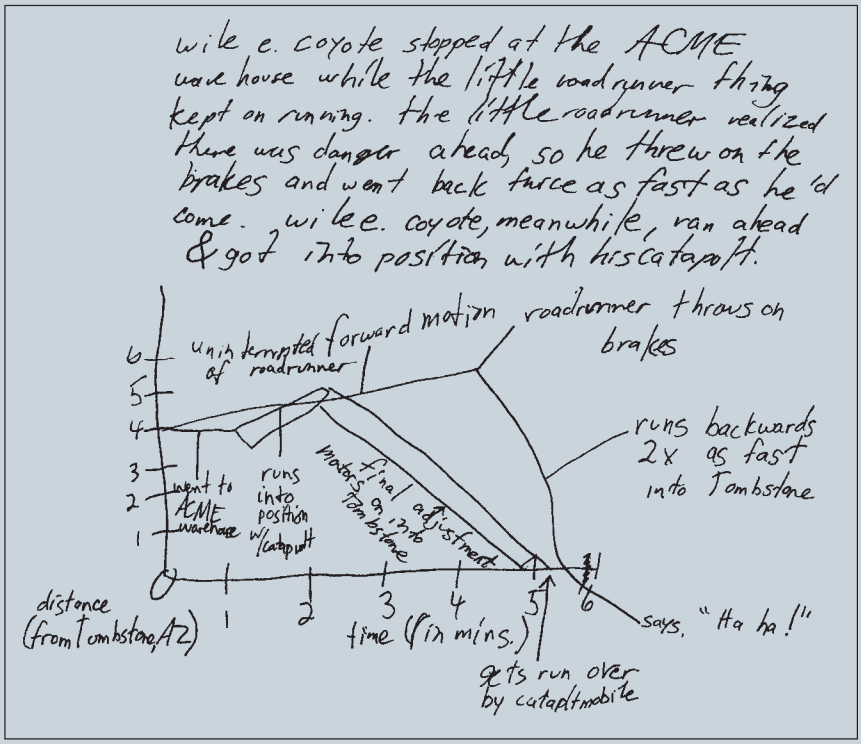
Fig. 3 Students' explanations of the What's Your Story? problem



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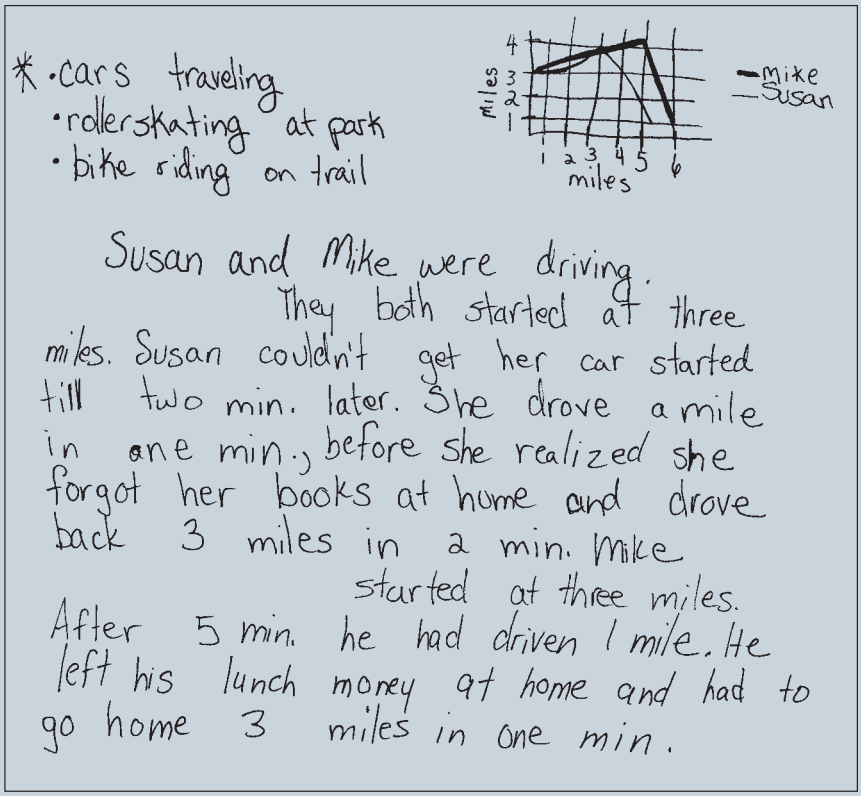

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