Solving Linear Equations and Inequalities: Solving Equations*

Denny Burzynski
Wade Ellis

Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter, the emphasis is on the mechanics of equation solving, which clearly explains how to isolate a variable. The goal is to help the student feel more comfortable with solving applied problems. Ample opportunity is provided for the student to practice translating words to symbols, which is an important part of the "Five-Step Method" of solving applied problems (discussed in modules (<link document="m21980"/>) and (<link document="m21979"/>)). Objectives of this module: be able to identify various types of equations, understand the meaning of solutions and equivalent equations, be able to solve equations of the form x + a = b and x - a = b, be familiar with and able to solve literal equations.

1 Overview

- Types of Equations
- Solutions and Equivalent Equations
- Literal Equations
- Solving Equations of the Form $x + a = b$ and $x - a = b$

2 Types of Equations

Identity
Some equations are always true. These equations are called identities. **Identities** are equations that are true for all acceptable values of the variable, that is, for all values in the domain of the equation.

$5x = 5x$ is true for all acceptable values of x.
$y + 1 = y + 1$ is true for all acceptable values of y.
$2 + 5 = 7$ is true, and no substitutions are necessary.
Contradiction
Some equations are never true. These equations are called contradictions. **Contradictions** are equations that are never true regardless of the value substituted for the variable.

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$x = x + 1$ is never true for any acceptable value of x.
$0 \cdot k = 14$ is never true for any acceptable value of k.
$2 = 1$ is never true.

Conditional Equation

The truth of some equations is conditional upon the value chosen for the variable. Such equations are called conditional equations. **Conditional equations** are equations that are true for at least one replacement of the variable and false for at least one replacement of the variable.

$x + 6 = 11$ is true only on the condition that $x = 5$.
$y - 7 = -1$ is true only on the condition that $y = 6$.

3 Solutions and Equivalent Equations

Solutions and Solving an Equation

The collection of values that make an equation true are called **solutions** of the equation. An equation is **solved** when all its solutions have been found.

Equivalent Equations

Some equations have precisely the same collection of solutions. Such equations are called **equivalent equations**. The equations

$$2x + 1 = 7, \quad 2x = 6 \quad \text{and} \quad x = 3$$

are equivalent equations because the only value that makes each one true is 3.

4 Sample Set A

Tell why each equation is an identity, a contradiction, or conditional.

Example 1

The equation $x - 4 = 6$ is a conditional equation since it will be true only on the condition that $x = 10$.

Example 2

The equation $x - 2 = x - 2$ is an identity since it is true for all values of x. For example,

$$\begin{array}{lllll} \text{if } x = 5, & 5 - 2 = 5 - 2 \text{ is true} \\ x = -7, & -7 - 2 = -7 - 2 \text{ is true} \end{array}$$

Example 3

The equation $a + 5 = a + 1$ is a contradiction since every value of a produces a false statement. For example,

$$\begin{array}{lllll} \text{if } a = 8, & 8 + 5 = 8 + 1 \text{ is false} \\ \text{if } a = -2, & -2 + 5 = -2 + 1 \text{ is false} \end{array}$$

5 Practice Set A

For each of the following equations, write "identity," "contradiction," or "conditional." If you can, find the solution by making an educated guess based on your knowledge of arithmetic.

Exercise 1 *(Solution on p. 9.)*

$x + 1 = 10$

Exercise 2 *(Solution on p. 9.)*

$y - 4 = 7$

Exercise 3 *(Solution on p. 9.)*
$5a = 25$

Exercise 4 *(Solution on p. 9.)*
$\frac{x}{4} = 9$

Exercise 5 *(Solution on p. 9.)*
$\frac{18}{b} = 6$

Exercise 6 *(Solution on p. 9.)*
$y - 2 = y - 2$

Exercise 7 *(Solution on p. 9.)*
$x + 4 = x - 3$

Exercise 8 *(Solution on p. 9.)*
$x + x + x = 3x$

Exercise 9 *(Solution on p. 9.)*
$8x = 0$

Exercise 10 *(Solution on p. 9.)*
$m - 7 = -5$

6 Literal Equations

Literal Equations

Some equations involve more than one variable. Such equations are called **literal equations**.

An equation is solved for a particular variable if that variable alone equals an expression that does not contain that particular variable.

The following equations are examples of literal equations.

1. $y = 2x + 7$. It is solved for y.
2. $d = rt$. It is solved for d.
3. $I = prt$. It is solved for I.
4. $z = \frac{x-u}{s}$. It is solved for z.
5. $y + 1 = x + 4$. This equation is not solved for any particular variable since no variable is isolated.

7 Solving Equation of the form $x + a = b$ and $x - a = b$

Recall that the equal sign of an equation indicates that the number represented by the expression on the left side is the same as the number represented by the expression on the right side.

This number	is the same as	this number
↓	↓	↓
x	$=$	6
$x + 2$	$=$	8
$x - 1$	$=$	5

This suggests the following procedures:

1. We can obtain an equivalent equation (an equation having the same solutions as the original equation) by **adding** the **same number** to **both sides** of the equation.

2. We can obtain an equivalent equation by **subtracting** the **same number** from **both sides** of the equation.

We can use these results to isolate x, thus solving for x.

Example 4: Solving $x + a = b$ for x

$x + a$	$=$	b	The a is associated with x by addition. Undo the association
$x + a - a$	$=$	$b - a$	by subtracting a from *both* sides.
$x + 0$	$=$	$b - a$	$a - a = 0$ and 0 is the additive identity. $x + 0 = x$.
x	$=$	$b - a$	This equation is equivalent to the first equation, and it is solved for x.

Example 5: Solving $x - a = b$ for x

$x - a$	$=$	b	The a is associated with x by subtraction. Undo the association
$x - a + a$	$=$	$b + a$	by adding a to *both* sides.
$x + 0$	$=$	$b + a$	$-a + a = 0$ and 0 is the additive identity. $x + 0 = x$.
x	$=$	$b + a$	This equation is equivalent to the first equation, and it is solved for x.

Example 6: Method for Solving $x + a = b$ and $x - a = b$ for x
To solve the equation $x + a = b$ for x, **subtract** a from **both** sides of the equation.
To solve the equation $x - a = b$ for x, **add** a to **both** sides of the equation.

8 Sample Set B

Example 7
Solve $x + 7 = 10$ for x.

$x + 7$	$=$	10	7 is associated with x by addition. Undo the association
$x + 7 - 7$	$=$	$10 - 7$	by subtracting 7 from *both* sides.
$x + 0$	$=$	3	$7 - 7 = 0$ and 0 is the additive identity. $x + 0 = x$.
x	$=$	3	x is isolated, and the equation $x = 3$ is equivalent to the original equation $x + 7 = 10$. Therefore, these two equation have the same solution. The solution to $x = 3$ is clearly 3. Thus, the solution to $x + 7 = 10$ is also 3.

Check: Substitute 3 for x in the original equation.

$x + 7$	$=$	10	
$3 + 7$	$=$	10	Is this correct?
10	$=$	10	Yes, this is correct.

Example 8
Solve $m - 2 = -9$ for m.

$$\begin{aligned} m-2 &= -9 && \text{2 is associated with } m \text{ by subtraction. Undo the association} \\ m-2+2 &= -9+2 && \text{by adding 2 from } \textit{both} \text{ sides.} \\ m+0 &= -7 && -2+2=0 \text{ and 0 is the additive identity. } m+0=m. \\ m &= -7 \end{aligned}$$

Check: Substitute -7 for m in the original equation.

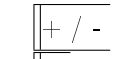
$$\begin{aligned} m-2 &= -9 \\ -7-2 &= -9 && \text{Is this correct?} \\ -9 &= -9 && \text{Yes, this is correct.} \end{aligned}$$

Example 9


Solve $y-2.181=-16.915$ for y.

$$\begin{aligned} y-2.181 &= -16.915 \\ y-2.181+2.181 &= -16.915+2.181 \\ y &= -14.734 \end{aligned}$$

On the Calculator

Type	16.915
Press	
Press	
Type	2.181
Press	
Display reads:	-14.734

Example 10

Solve $y+m=s$ for y.

$$\begin{aligned} y+m &= s && m \text{ is associated with } y \text{ by addition. Undo the association} \\ y+m-m &= s-m && \text{by subtracting } m \text{ from } \textit{both} \text{ sides.} \\ y+0 &= s-m && m-m=0 \text{ and 0 is the additive identity. } y+0=y. \\ y &= s-m \end{aligned}$$

Check: Substitute $s-m$ for y in the original equation.

$$\begin{aligned} y+m &= s \\ s-m+m &= s && \text{Is this correct?} \\ s &= s && \text{True} \quad \text{Yes, this is correct.} \end{aligned}$$

Example 11

Solve $k-3h=-8h+5$ for k.

$$\begin{aligned} k-3h &= -8h+5 && 3h \text{ is associated with } k \text{ by subtraction. Undo the association} \\ k-3h+3h &= -8h+5+3h && \text{by adding } 3h \text{ to } \textit{both} \text{ sides.} \\ k+0 &= -5h+5 && -3h+3h=0 \text{ and 0 is the additive identity. } k+0=k. \\ k &= -5h+5 \end{aligned}$$

9 Practice Set B

Exercise 11 *(Solution on p. 9.)*
Solve $y - 3 = 8$ for y.

Exercise 12 *(Solution on p. 9.)*
Solve $x + 9 = -4$ for x.

Exercise 13 *(Solution on p. 9.)*
Solve $m + 6 = 0$ for m.

Exercise 14 *(Solution on p. 9.)*
Solve $g - 7.2 = 1.3$ for g.

Exercise 15 *(Solution on p. 9.)*
solve $f + 2d = 5d$ for f.

Exercise 16 *(Solution on p. 9.)*
Solve $x + 8y = 2y - 1$ for x.

Exercise 17 *(Solution on p. 9.)*
Solve $y + 4x - 1 = 5x + 8$ for y.

10 Exercises

For the following problems, classify each of the equations as an identity, contradiction, or conditional equation.

Exercise 18 *(Solution on p. 9.)*
$m + 6 = 15$

Exercise 19
$y - 8 = -12$

Exercise 20 *(Solution on p. 9.)*
$x + 1 = x + 1$

Exercise 21
$k - 2 = k - 3$

Exercise 22 *(Solution on p. 9.)*
$g + g + g + g = 4g$

Exercise 23
$x + 1 = 0$

For the following problems, determine which of the literal equations have been solved for a variable. Write "solved" or "not solved."

Exercise 24 *(Solution on p. 9.)*
$y = 3x + 7$

Exercise 25
$m = 2k + n - 1$

Exercise 26 *(Solution on p. 9.)*
$4a = y - 6$

Exercise 27
$hk = 2k + h$

Exercise 28 *(Solution on p. 9.)*
$2a = a + 1$

Exercise 29
$5m = 2m - 7$

Exercise 30 *(Solution on p. 9.)*
$m = m$

For the following problems, solve each of the conditional equations.

Exercise 31
$h - 8 = 14$

Exercise 32 *(Solution on p. 9.)*
$k + 10 = 1$

Exercise 33
$m - 2 = 5$

Exercise 34 *(Solution on p. 10.)*
$y + 6 = -11$

Exercise 35
$y - 8 = -1$

Exercise 36 *(Solution on p. 10.)*
$x + 14 = 0$

Exercise 37
$m - 12 = 0$

Exercise 38 *(Solution on p. 10.)*
$g + 164 = -123$

Exercise 39
$h - 265 = -547$

Exercise 40 *(Solution on p. 10.)*
$x + 17 = -426$

Exercise 41
$h - 4.82 = -3.56$

Exercise 42 *(Solution on p. 10.)*
$y + 17.003 = -1.056$

Exercise 43
$k + 1.0135 = -6.0032$

Exercise 44 *(Solution on p. 10.)*
Solve $n + m = 4$ for n.

Exercise 45
Solve $P + 3Q - 8 = 0$ for P.

Exercise 46 *(Solution on p. 10.)*
Solve $a + b - 3c = d - 2f$ for b.

Exercise 47
Solve $x - 3y + 5z + 1 = 2y - 7z + 8$ for x.

Exercise 48 *(Solution on p. 10.)*
Solve $4a - 2b + c + 11 = 6a - 5b$ for c.

11 Exercises for Review

Exercise 49
(**here**[1]) Simplify $\left(4x^5y^2\right)^3$.

Exercise 50 ***(Solution on p. 10.)***
(**here**[2]) Write $\frac{20x^3y^7}{5x^5y^3}$ so that only positive exponents appear.

Exercise 51
(**here**[3]) Write the number of terms that appear in the expression $5x^2+2x-6+(a+b)$, and then list them.

Exercise 52 ***(Solution on p. 10.)***
(**here**[4]) Find the product. $(3x-1)^2$.

Exercise 53
(**here**[5]) Specify the domain of the equation $y=\frac{5}{x-2}$.

[1] "Basic Properties of Real Numbers: The Power Rules for Exponents" <http://cnx.org/content/m21897/latest/>
[2] "Basic Operations with Real Numbers: Negative Exponents" <http://cnx.org/content/m21882/latest/>
[3] "Algebraic Expressions and Equations: Algebraic Expressions" <http://cnx.org/content/m18875/latest/>
[4] "Algebraic Expressions and Equations: Special Binomial Products" <http://cnx.org/content/m21858/latest/>
[5] "Algebraic Expressions and Equations: Terminology Associated with Equations" <http://cnx.org/content/m21849/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 2)
conditional, $x = 9$
Solution to Exercise (p. 2)
conditional, $y = 11$
Solution to Exercise (p. 2)
conditional, $a = 5$
Solution to Exercise (p. 3)
conditional, $x = 36$
Solution to Exercise (p. 3)
conditional, $b = 3$
Solution to Exercise (p. 3)
identity
Solution to Exercise (p. 3)
contradiction
Solution to Exercise (p. 3)
identity
Solution to Exercise (p. 3)
conditional, $x = 0$
Solution to Exercise (p. 3)
conditional, $m = 2$
Solution to Exercise (p. 6)
$y = 11$
Solution to Exercise (p. 6)
$x = -13$
Solution to Exercise (p. 6)
$m = -6$
Solution to Exercise (p. 6)
$g = 8.5$
Solution to Exercise (p. 6)
$f = 3d$
Solution to Exercise (p. 6)
$x = -6y - 1$
Solution to Exercise (p. 6)
$y = x + 9$
Solution to Exercise (p. 6)
conditional
Solution to Exercise (p. 6)
identity
Solution to Exercise (p. 6)
identity
Solution to Exercise (p. 6)
solved
Solution to Exercise (p. 6)
not solved
Solution to Exercise (p. 6)
not solved
Solution to Exercise (p. 7)
not solved

Solution to Exercise (p. 7)
$k = -9$
Solution to Exercise (p. 7)
$y = -17$
Solution to Exercise (p. 7)
$x = -14$
Solution to Exercise (p. 7)
$g = -287$
Solution to Exercise (p. 7)
$x = -443$
Solution to Exercise (p. 7)
$y = -18.059$
Solution to Exercise (p. 7)
$n = 4 - m$
Solution to Exercise (p. 7)
$b = -a + 3c + d - 2f$
Solution to Exercise (p. 7)
$c = 2a - 3b - 11$
Solution to Exercise (p. 8)
$\frac{4y^4}{x^2}$
Solution to Exercise (p. 8)
$9x^2 - 6x + 1$