

EQUATIONS AND INEQUALITIES: SOLVING LINEAR EQUATIONS*

Free High School Science Texts Project

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1 Strategy for Solving Equations

This chapter is all about solving different types of equations for one or two variables. In general, we want to get the unknown variable alone on the left hand side of the equation with all the constants on the right hand side of the equation. For example, in the equation $x - 1 = 0$, we want to be able to write the equation as $x = 1$.

As we saw in review of past work¹ (section on rearranging equations), an equation is like a set of weighing scales that must always be balanced. When we solve equations, we need to keep in mind that what is done to one side must be done to the other.

1.1 Method: Rearranging Equations

You can add, subtract, multiply or divide both sides of an equation by any number you want, as long as you always do it to both sides.

For example, in the equation $x + 5 - 1 = -6$, we want to get x alone on the left hand side of the equation. This means we need to subtract 5 and add 1 on the left hand side. However, because we need to keep the equation balanced, we also need to subtract 5 and add 1 on the right hand side.

$$\begin{aligned}
 x + 5 - 1 &= -6 \\
 x + 5 - 5 - 1 + 1 &= -6 - 5 + 1 \\
 x + 0 + 0 &= -11 + 1 \\
 x &= -10
 \end{aligned}
 \tag{1}$$

In another example, $\frac{2}{3}x = 8$, we must divide by 2 and multiply by 3 on the left hand side in order to get x alone. However, in order to keep the equation balanced, we must also divide by 2 and multiply by 3 on the

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¹"Review of Past Work" <<http://cnx.org/content/m31330/latest/>>

right hand side.

$$\begin{aligned}
 \frac{2}{3}x &= 8 \\
 \frac{2}{3}x \div 2 \times 3 &= 8 \div 2 \times 3 \\
 \frac{2}{2} \times \frac{3}{3} \times x &= \frac{8 \times 3}{2} \\
 1 \times 1 \times x &= 12 \\
 x &= 12
 \end{aligned}
 \tag{2}$$

These are the basic rules to apply when simplifying any equation. In most cases, these rules have to be applied more than once, before we have the unknown variable on the left hand side of the equation.

TIP: The following must also be kept in mind:

1. Division by 0 is undefined.
2. If $\frac{x}{y} = 0$, then $x = 0$ and $y \neq 0$, because division by 0 is undefined.

We are now ready to solve some equations!

1.1.1 Investigation : Strategy for Solving Equations

In the following, identify what is wrong.

$$\begin{aligned}
 4x - 8 &= 3(x - 2) \\
 4(x - 2) &= 3(x - 2) \\
 \frac{4(x-2)}{(x-2)} &= \frac{3(x-2)}{(x-2)} \\
 4 &= 3
 \end{aligned}
 \tag{3}$$

2 Solving Linear Equations

The simplest equation to solve is a linear equation. A linear equation is an equation where the power of the variable(letter, e.g. x) is 1(one). The following are examples of linear equations.

$$\begin{aligned}
 2x + 2 &= 1 \\
 \frac{2-x}{3x+1} &= 2 \\
 \frac{4}{3}x - 6 &= 7x + 2
 \end{aligned}
 \tag{4}$$

In this section, we will learn how to find the value of the variable that makes both sides of the linear equation true. For example, what value of x makes both sides of the very simple equation, $x + 1 = 1$ true.

Since the definition of a linear equation is that if the variable has a highest power of one (1), there is at most *one solution* or *root* for the equation.

This section relies on all the methods we have already discussed: multiplying out expressions, grouping terms and factorisation. Make sure that you are comfortable with these methods, before trying out the work in the rest of this chapter.

$$\begin{aligned}
 2x + 2 &= 1 \\
 2x &= 1 - 2 \quad (\text{liketermstogether}) \\
 2x &= -1 \quad (\text{simplifiedasmuchaspossible})
 \end{aligned}
 \tag{5}$$

Now we see that $2x = -1$. This means if we divide both sides by 2, we will get:

$$x = -\frac{1}{2} \quad (6)$$

If we substitute $x = -\frac{1}{2}$, back into the original equation, we get:

$$\begin{aligned} LHS &= 2x + 2 \\ &= 2\left(-\frac{1}{2}\right) + 2 \\ &= -1 + 2 \\ &= 1 \end{aligned} \quad (7)$$

and

$$RHS = 1$$

That is all that there is to solving linear equations.

TIP: Solving Equations

When you have found the solution to an equation, substitute the solution into the original equation, to check your answer.

2.1 Method: Solving Equations

The general steps to solve equations are:

1. Expand (Remove) all brackets.
2. "Move" all terms with the variable to the left hand side of the equation, and all constant terms (the numbers) to the right hand side of the equals sign. Bearing in mind that the sign of the terms will change from (+) to (-) or vice versa, as they "cross over" the equals sign.
3. Group all like terms together and simplify as much as possible.
4. Factorise if necessary.
5. Find the solution.
6. Substitute solution into **original** equation to check answer.

Khan academy video on equations - 1

This media object is a Flash object. Please view or download it at
<<http://www.youtube.com/v/f15zA0PhSek&rel=0>>

Figure 1

Exercise 1: Solving Linear Equations (Solution on p. 5.)

Solve for x : $4 - x = 4$

Exercise 2: Solving Linear Equations (Solution on p. 5.)

Solve for x : $4(2x - 9) - 4x = 4 - 6x$

Exercise 3: Solving Linear Equations (Solution on p. 6.)

Solve for x : $\frac{2-x}{3x+1} = 2$

Exercise 4: Solving Linear Equations (Solution on p. 6.)

Solve for x : $\frac{4}{3}x - 6 = 7x + 2$

2.1.1 Solving Linear Equations

1. Solve for y : $2y - 3 = 7$ Click here for the solution²
2. Solve for w : $-3w = 0$ Click here for the solution³
3. Solve for z : $4z = 16$ Click here for the solution⁴
4. Solve for t : $12t + 0 = 144$ Click here for the solution⁵
5. Solve for x : $7 + 5x = 62$ Click here for the solution⁶
6. Solve for y : $55 = 5y + \frac{3}{4}$ Click here for the solution⁷
7. Solve for z : $5z = 3z + 45$ Click here for the solution⁸
8. Solve for a : $23a - 12 = 6 + 2a$ Click here for the solution⁹
9. Solve for b : $12 - 6b + 34b = 2b - 24 - 64$ Click here for the solution¹⁰
10. Solve for c : $6c + 3c = 4 - 5(2c - 3)$ Click here for the solution¹¹
11. Solve for p : $18 - 2p = p + 9$ Click here for the solution¹²
12. Solve for q : $\frac{4}{q} = \frac{16}{24}$ Click here for the solution¹³
13. Solve for q : $\frac{4}{1} = \frac{q}{2}$ Click here for the solution¹⁴
14. Solve for r : $-(-16 - r) = 13r - 1$ Click here for the solution¹⁵
15. Solve for d : $6d - 2 + 2d = -2 + 4d + 8$ Click here for the solution¹⁶
16. Solve for f : $3f - 10 = 10$ Click here for the solution¹⁷
17. Solve for v : $3v + 16 = 4v - 10$ Click here for the solution¹⁸
18. Solve for k : $10k + 5 + 0 = -2k + -3k + 80$ Click here for the solution¹⁹
19. Solve for j : $8(j - 4) = 5(j - 4)$ Click here for the solution²⁰
20. Solve for m : $6 = 6(m + 7) + 5m$ Click here for the solution²¹

²See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcR>>

³See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcR>>

⁴See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcR>>

⁵See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcR>>

⁶See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcR>>

⁷See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcN>>

⁸See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcN>>

⁹See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcN>>

¹⁰See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcN>>

¹¹See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcQ>>

¹²See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcQ>>

¹³See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcQ>>

¹⁴See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcQ>>

¹⁵See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcQ>>

¹⁶See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcU>>

¹⁷See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcU>>

¹⁸See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcU>>

¹⁹See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcU>>

²⁰See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcU>>

²¹See the file at <<http://cnx.org/content/m39780/latest/http://www.fhsst.org/lcU>>

Solutions to Exercises in this Module

Solution to Exercise (p. 3)

Step 1. We are given $4 - x = 4$ and are required to solve for x .

Step 2. Since there are no brackets, we can start with grouping like terms and then simplifying.

Step 3.

$$\begin{aligned}
 4 - x &= && 4 \\
 -x &= 4 - 4 && \text{(move all constant terms (numbers) to the RHS (right hand side))} \\
 -x &= && 0 && \text{(group like terms together)} \\
 -x &= && 0 && \text{(simplify grouped terms)} \\
 -x &= && 0 \\
 \therefore x &= && 0
 \end{aligned} \tag{8}$$

Step 4. Substitute solution into original equation:

$$4 - 0 = 4 \tag{9}$$

$$4 = 4 \tag{10}$$

Since both sides are equal, the answer is correct.

Step 5. The solution of $4 - x = 4$ is $x = 0$.

Solution to Exercise (p. 3)

Step 1. We are given $4(2x - 9) - 4x = 4 - 6x$ and are required to solve for x .

Step 2. We start with expanding the brackets, then grouping like terms and then simplifying.

Step 3.

$$\begin{aligned}
 4(2x - 9) - 4x &= && 4 - 6x \\
 8x - 36 - 4x &= && 4 - 6x && \text{(expand the brackets)} \\
 8x - 4x + 6x &= && 4 + 36 && \text{move all terms with } x \text{ to the LHS} \\
 \text{and all constant terms to the RHS of the} &= && && \\
 (8x - 4x + 6x) &= && (4 + 36) && \text{(group like terms together)} \\
 10x &= && 40 && \text{(simplify grouped terms)} \\
 \frac{10}{10}x &= && \frac{40}{10} && \text{(divide both sides by 10)} \\
 x &= && 4
 \end{aligned} \tag{11}$$

Step 4. Substitute solution into original equation:

$$\begin{aligned}
 4(2(4) - 9) - 4(4) &= && 4 - 6(4) \\
 4(8 - 9) - 16 &= && 4 - 24 \\
 4(-1) - 16 &= && -20 \\
 -4 - 16 &= && -20 \\
 -20 &= && -20
 \end{aligned} \tag{12}$$

Since both sides are equal to -20 , the answer is correct.

Step 5. The solution of $4(2x - 9) - 4x = 4 - 6x$ is $x = 4$.

Solution to Exercise (p. 3)

Step 1. We are given $\frac{2-x}{3x+1} = 2$ and are required to solve for x .

Step 2. Since there is a denominator of $(3x + 1)$, we can start by multiplying both sides of the equation by $(3x + 1)$. But because division by 0 is not permissible, there is a restriction on a value for x . ($x \neq \frac{-1}{3}$)

Step 3.

$$\begin{aligned} \frac{2-x}{3x+1} &= 2 \\ (2-x) &= 2(3x+1) \\ 2-x &= 6x+2 \quad (\text{remove/expand brackets}) \\ -x-6x &= 2-2 \quad \text{move all terms containing } x \text{ to the LHS} \\ &\text{and all constant terms (numbers) to the RHS.} \end{aligned} \tag{13}$$

$$\begin{aligned} -7x &= 0 \quad (\text{simplify grouped terms}) \\ x &= 0 \div (-7) \\ \text{therefore } x &= 0 \quad \text{zero divided by any number is 0} \end{aligned}$$

Step 4. Substitute solution into original equation:

$$\begin{aligned} \frac{2-(0)}{3(0)+1} &= 2 \\ \frac{2}{1} &= 2 \end{aligned} \tag{14}$$

Since both sides are equal to 2, the answer is correct.

Step 5. The solution of $\frac{2-x}{3x+1} = 2$ is $x = 0$.

Solution to Exercise (p. 3)

Step 1. We are given $\frac{4}{3}x - 6 = 7x + 2$ and are required to solve for x .

Step 2. We start with multiplying each of the terms in the equation by 3, then grouping like terms and then simplifying.

Step 3.

$$\begin{aligned} \frac{4}{3}x - 6 &= 7x + 2 \\ 4x - 18 &= 21x + 6 \quad (\text{each term is multiplied by 3}) \\ 4x - 21x &= 6 + 18 \quad (\text{move all terms with } x \text{ to the LHS} \\ &\text{and all constant terms to the RHS of the } = \\ -17x &= 24 \quad (\text{simplify grouped terms}) \\ \frac{-17}{-17}x &= \frac{24}{-17} \quad (\text{divide both sides by } -17) \\ x &= \frac{-24}{17} \end{aligned} \tag{15}$$

Step 4. Substitute solution into original equation:

$$\begin{aligned} \frac{4}{3} \times \frac{-24}{17} - 6 &= 7 \times \frac{-24}{17} + 2 \\ \frac{4 \times (-8)}{(17)} - 6 &= \frac{7 \times (-24)}{17} + 2 \\ \frac{(-32)}{17} - 6 &= \frac{-168}{17} + 2 \\ \frac{-32-102}{17} &= \frac{(-168)+34}{17} \\ \frac{-134}{17} &= \frac{-134}{17} \end{aligned} \tag{16}$$

Since both sides are equal to $\frac{-134}{17}$, the answer is correct.

Step 5. The solution of $\frac{4}{3}x - 6 = 7x + 2$ is, $x = \frac{-24}{17}$.