UNIT 3

Optimization Problems
(ch.3 in text)
L1 (3.1) Position, Velocity, and Acceleration

Notation:

- $s(t)$ or $h(t)$: position
- $s'(t) = v(t)$: velocity
- $s''(t) = v'(t) = a(t)$: acceleration

Read Summary on p. 126
Ex1: The function \( s(t) = t(t - 3)^2 \) describes the position of an object moving along a straight line, \( t \geq 0 \), in metres at time \( t \), in seconds.

a) Calculate the velocity and acceleration at any time \( t \).

b) Find the velocity and acceleration at 4 seconds.

c) Determine whether the object is moving in a positive or negative direction at time \( t = 4 \) seconds.

\[
\begin{align*}
S'(t) &= v(t) = (t-3)^2 + 2(t-3)(t) \\
&= t^2 - 6t + 9 + 2t^2 - 6t \\
&= 3t^2 - 12t + 9 \\
S''(t) &= v'(t) = a(t) = 6t - 12
\end{align*}
\]

b) \( v(4) = 3(4)^2 - 12(4) + 9 = 9 \)

\( a(4) = 6(4) - 12 = 12 \)

\[\therefore \text{ The velocity \& acceleration at 4 seconds is } 9 \frac{m}{s} \text{ FWD \& } 12 \frac{m}{s^2} \text{ FWD respectively.}\]

c) Since velocity is positive we are moving in a positive direction.
Ex2: p.128 #10

\[ s(t) = t^\frac{5}{2}(7-t), t \geq 0 \]

a) \[ v(t) = \frac{5}{2} t^\frac{3}{2}(7-t) - t^\frac{3}{2} \]
   \[ = \frac{35}{2} t^\frac{3}{2} - \frac{5}{2} t^\frac{3}{2} - t^\frac{3}{2} \]
   \[ = \frac{25}{2} t^\frac{3}{2} - \frac{7}{2} t^\frac{3}{2} \]
   \[ a(t) = \frac{105}{4} t^\frac{1}{2} - \frac{35}{4} t^\frac{1}{2} \]

b) Set \[ v(t) = 0 \] to solve for \( t \).
   \[ 0 = \frac{25}{2} t^\frac{3}{2} - \frac{7}{2} t^\frac{3}{2} \]
   \[ 0 = \frac{25}{2} t^\frac{3}{2} \left[ 5 - t \right] \]
   \[ t = 0 \text{ or } 5 \]

i. The object is stopped at 5 seconds.

5 seconds (we 2nd deriv test if you need prof) \( a(t) < 0 \) max.

ii. [Diagram showing acceleration vs. time]
   \[ a(t) = \frac{105}{4} t^\frac{1}{2} - \frac{35}{4} t^\frac{1}{2} \]
   Set \[ a(t) = 0 \] to determine inflection pts.
   \[ 0 = \frac{25}{2} t^\frac{3}{2} \left[ 3 - t \right] \]
   \[ t = 0 \text{ or } 3 \]

   \[ \text{Points: } (2.7, -2.7) \text{ or } (3, 0) \]
   \[ a(t) \quad + \\quad - \]

i. The object has positive acceleration between 0.6 to 3 seconds.

\[ s(t) = 0 \] when object is returned to original pos'n.
   \[ 0 = t^\frac{5}{2}(7-t) \]
   \[ t = \frac{14}{5} \text{ or } 7 \]

i. The object returns to starting position at 7 seconds.
Assigned Work:

p.127-129 #4, 5, 6c, 8, 9, 11, 12, 13b (read example 4 on p.124), 16