# UNIT 3

# Optimization Problems (ch.3 in text)

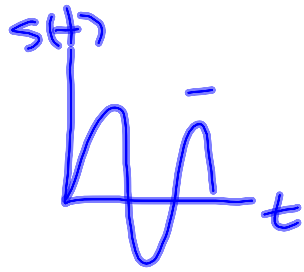
# L1 (3.1) Position, Velocity, and Acceleration

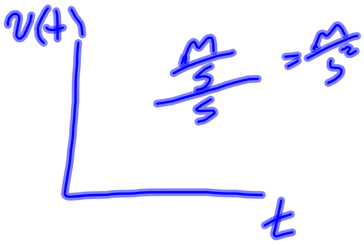
**Notation:**

s(t) or h(t): postion

s'(t) = v(t): velocity

s''(t) = v'(t) = a(t): acceleration




Read Summary on p. 126

Ex1: The function $s(t) = t(t - 3)^2$ describes the position of an object moving along a straight line, $t \geq 0$, in metres at time t, in seconds.

a) Calculate the velocity and acceleration at any time t.

b) Find the velocity and acceleration at 4 seconds.

c) Determine whether the object is moving in a positive or negative direction at time t = 4 seconds.

$(s \cdot v) \leftarrow$ sign

a) $s'(t) = v(t) = (t-3)^2 + 2(t-3)(t)$

$= t^2 - 6t + 9 + 2t^2 - 6t$

$= 3t^2 - 12t + 9$

$s''(t) = v'(t) = a(t) = 6t - 12$

b) $v(4) = 3(4)^2 - 12(4) + 9$

$= 9$

$a(4) = 6(4) - 12$

$= 12$

$\therefore$ The velocity & acceleration @ 4 seconds is $9\ \frac{m}{s}$ FWD & $12\ \frac{m}{s^2}$ FWD respectively.

c) Since velocity is positive we are moving in a positive direction.

Ex2: p.128 #10 $s(t) = t^{\frac{5}{2}}(7-t)$, $t \geq 0$

a) $v(t) = \frac{5}{2}t^{\frac{3}{2}}(7-t) - t^{\frac{5}{2}}$

$= \frac{35}{2}t^{3/2} - \frac{5}{2}t^{5/2} - t^{5/2}$

$= \frac{35}{2}t^{\frac{3}{2}} - \frac{7}{2}t^{\frac{5}{2}}$

$a(t) = \frac{105}{4}t^{1/2} - \frac{35}{4}t^{3/2}$

b) Set $v(t) = 0$ to solve for $t$.

$0 = \frac{35}{2}t^{3/2} - \frac{7}{2}t^{\frac{5}{2}}$

$0 = \frac{7}{2}t^{3/2}\left[5 - t\right]$

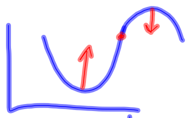
$t = 0$ or $5$

$\therefore$ The object is stopped at 5 seconds and 0 seconds.

c) 5 seconds (use 2nd deriv test if you need proof) $a(5) < 0$

$\therefore$ MAX.

d)



acceleration ~ concavity

$a(t) = \frac{105}{4}t^{1/2} - \frac{35}{4}t^{3/2}$

Set $a(t) = 0$ to determine inflection pts.

$0 = \frac{35}{4}t^{1/2}\left[3 - t\right]$

$t = 0$ or $3$

| Intervals | $0 < t < 3$ | $t > 3$ |
|---|---|---|
| $a(t)$ | $+$ | $-$ |

$\therefore$ The object has positive acceleration between $0<t<3$ seconds.

e) $s(t) = 0$ when object is returned to original pos'n.

$0 = t^{\frac{5}{2}}(7-t)$

$t = \cancel{0}$ or $7$

$\therefore$ The object returns to starting position at 7 seconds.

Assigned Work:

p.127-129 #4, 5, 6c, 8, 9,
11, 12,
13b (read example 4 on p.124),
16