13b) \( S(t) = t^3 - 12t - 9 \)

\( S'(t) = v(t) = 3t^2 - 12 \)

\( S''(t) = a(t) = 6t \)

Set \( S'(t) = 0 \)

0 = 3t^2 - 12

\( t = \pm 2 \quad t > 0 \)

\( t = 2 \)

\( t = 0 \quad S = -9 \)

\( t = 2 \quad S = 25 \)
\[
\begin{align*}
    s(t) &= \begin{cases} 
    0, & t < 0 \\
    \frac{t^3}{t^2 + 1}, & t \geq 0 
    \end{cases} \\
    v(t) &= \frac{3t^2(t^2 - 1) - 2t(t^3)}{(t^2 + 1)^2}, \quad t \geq 0 \\
    &= \frac{3t^4 + 3t^2 - 2t^4}{(t^2 + 1)^2} \\
    &= \frac{t^4 + 3t^2}{(t^2 + 1)^2} \\
    a(t) &= \frac{(4t^3 + 6t)(t^2 + 1)^2 - 2(4t^2 + 1)(2t)(t^4 + 3t^2)}{(t^2 + 1)^4} \\
    a(0) &= 0 \checkmark
\end{align*}
\]
L2 (3.2) Max & Min Values on an Interval

The following is a section of quartic polynomial function.

a) Identify the absolute max and min points.

b) Identify the local max & min points.
To find the max & min values (a.k.a. extreme values)

1- Solve $f'(x) = 0$ and find all points in the given interval.

2- Evaluate $f(x)$ at the endpoints.

3- Compare the y-values found in steps 1 & 2.
Ex1: Find all local and extreme max & min values of the function on the given interval.

\[ f(x) = 3x^4 - 4x^3 - 36x^2 + 20, \ x \in [-3, 4] \]

\[ f'(x) = 12x^3 - 12x^2 - 72x, \ x \in [-3, 4] \]

Set \( f'(x) = 0 \)

\( 0 = 12x^3 - 12x^2 - 72x \)
\( 0 = 12x(x^2 - x - 6) \)
\( 0 = 12x(x - 3)(x + 2) \)
\( x = 0, 3, -2 \)

\( f(0) = 20 \)
\( f(3) = -169 \quad \text{extreme min.} \)
\( f(-2) = -44 \)

To get endpoints use the info from interval.

\( f(-3) = 47 \quad \text{extreme max.} \)
\( f(4) = -44 \)
Ex2: The amount of light intensity on a point is given by the function 

\[ I(t) = \frac{t^2 + 2t + 16}{t + 2} \]

where \( t \) is the time in seconds and \( t \in [0, 14] \).

a) Determine the time of minimal intensity.
b) What is the minimal intensity?

\[ I(0) = 8 \]
\[ I(14) = 15 \]

Find endpoints:

\[ I'(t) = \frac{(2t + 2)(t + 2) - t^2 - 2t - 16}{(t + 2)^2} \]

\[ I'(t) = \frac{2t^2 + 4t + 4 - t^2 - 2t - 16}{(t + 2)^2} \]
\[ I'(t) = \frac{t^2 + 2t - 12}{(t + 2)^2} \]
\[ I'(t) = \frac{(t + 6)(t - 2)}{(t + 2)^2} \]

Set \( I'(t) = 0 \)

\[ 0 = \frac{(t + 6)(t - 2)}{(t + 2)^2} \]

\[ t = -6, 2 \] but \( t \in [0, 14] \)

So \( t = 2 \)

\[ I(2) = 6 \]

\[ \ldots \text{ After comparing intensities for all local minima, the lowest intensity is 6 units which occurs at 2 seconds.} \]

Think about \( \frac{x^2 + 5}{x - 2}, x \in [0, 15] \), \( x = 2 \)
Assigned Work:

p136-138 # 2ab, 3d, 4ab, 6, 7a, 8, 9