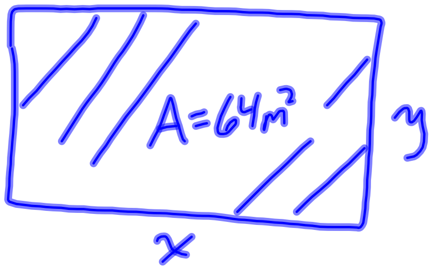
L4 (3.3) Optimizing Distance

Assigned Work:

p. 147 #15, 16, 20 + Handout #1 to 5, 8, 9

p. 145 #6



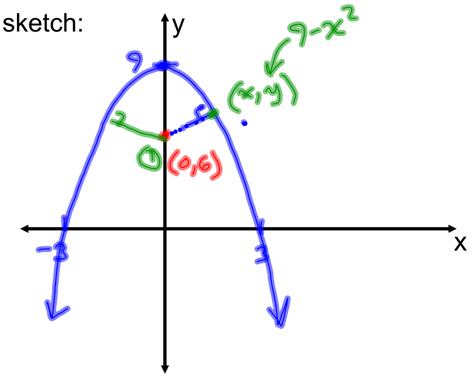
Let x & y rep the dimensions of the smallest rectangle (perimeter).

Eq. 1	Eq. 2
$P = 2l + 2w$	$A = lw$
$P_{(x)} = 2x + 2\left(\frac{64}{x}\right)$	$64 = xy$
	$y = \frac{64}{x}$

Ex1: Find the minimal distance from the point (0, 6) to the curve $y = 9 - x^2$. Discuss how to find the coordinate of the point on the curve that is closest to (0, 6)

b

sketch:



Eq.2 $y = 9 - x^2$

Eq.1

$$d(x) = \left[(x-0)^2 + (9-x^2-6)^2\right]^{\frac{1}{2}}$$

Recal

$$d'(x) = \frac{1}{\cancel{2}}\left[x^2 + (-x^2+3)^2\right]^{-\frac{1}{2}}\left(\cancel{2}x + \cancel{2}(-x^2+3)(-2x)\right)$$

$$= \frac{x + 2x^3 - 6x}{\left[x^2 + (-x^2+3)^2\right]^{\frac{1}{2}}}$$

$2x^3 - 5x$

$$= \frac{x(2x^2-5)}{\left[x^2 + (3-x^2)^2\right]^{\frac{1}{2}}}$$

Set $d'(x) = 0$ to find critical values:

$0 = x(2x^2-5)$

$x = 0, \pm\sqrt{\frac{5}{2}}$

check*

$d(0) = 3$ $\quad d\left(\pm\sqrt{\frac{5}{2}}\right) = \frac{\sqrt{11}}{2} \doteq 1.66$

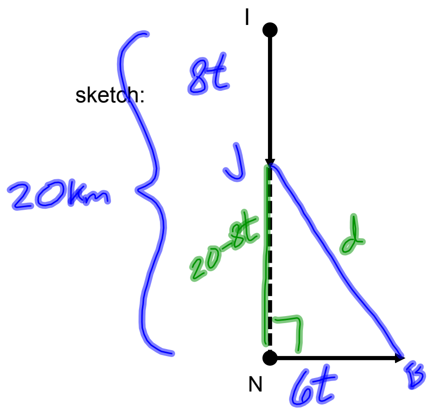
$\therefore$ The minimal distance to the curve $y = 9 - x^2$ from point (0,6) is $\frac{\sqrt{11}}{2}$ or approx 1.66 units.

Ex2: p. 143 EXAMPLE 3

Ian's house is located 20 km north of Nada's house. One morning, Ian leaves his house and jogs south at a 8km/h. At the same time, Nada leaves her house and jogs east at 6km/h.
If they both run for 2.5 hours, when will Ian and Nada be closest together?

Let t rep the time when Ian and Nada will be closest together.

sketch:



$$d(t) = \left((20-8t)^2 + (6t)^2\right)^{\frac{1}{2}}$$

$$d'(t) = \frac{1}{\cancel{2}}\left((20-8t)^2 + 36t^2\right)^{-\frac{1}{2}}\left(\cancel{2}(20-8t)(-8) + \cancel{72}^{36}t\right)$$

$$= \frac{64t + 36t - 160}{\left((20-8t)^2 + 36t^2\right)^{1/2}}$$

$$= \frac{100t - 160}{\left((20-8t)^2 + 36t^2\right)^{1/2}}$$

Set $d'(t) = 0$

$0 = 100t - 160$

$t = 1.6$

$d(1.6) = 12$

$d(0) = 20$

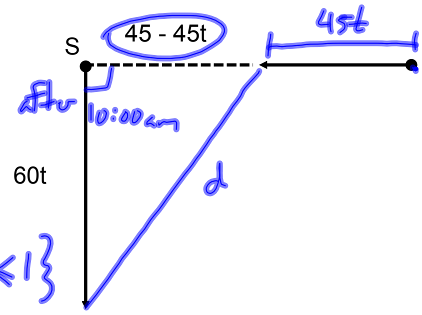
$d(2.5) = 15$

$\therefore$ Nada & Ian are closest together after 1.6 hours.

Ex3: p. 147 #15

A train leaves the station at 10:00 a.m. and travels due south at 60 km/h. Another train has been heading due west at 45 km/h and reaches the same station at 11:00 a.m.
At what time of day (in hours and minutes) were the two trains closest to each other?

sketch:



Let t rep. the time after 10:00am at which the trains are closest together.

$\{t \in \mathbb{R},\ 0 \le t \le 1\}$

$$d(t) = \left[(60t)^2 + (45-45t)^2\right]^{1/2}$$

$$= \left[3600t^2 + 2025 - 4050t + 2025t^2\right]^{1/2}$$

$$= \left[5625t^2 - 4050t + 2025\right]^{1/2}$$

$$d'(t) = \frac{1}{2}\left(5625t^2 - 4050t + 2025\right)^{-1/2}\left(11250t - 4050\right)$$

$$= \frac{5625t - 2025}{\left(5625t^2 - 4050t + 2025\right)^{1/2}}$$

Set $d'(t) = 0$

$$0 = 5625t - 2025$$

$$t = 0.36$$

Check distances at endpoints & at $t = 0.36$;

$d(0.36) = 36$

$d(0) = 45$

$d(1) = 60$

Time of day $= 0.36 \cancel{h} \times \frac{60 \text{ minutes}}{1 \cancel{h}} + 10{:}$

$= 21.6_{\text{min}} + 10{:}00$

$= 10{:}22$

$\therefore$ At approx. 10:22 a.m. the