

L5 (4.5) The Curve Sketching Algorithm

p.207

The Algorithm for Curve Sketching:

- ✓ • Domain ← Discontinuities?
- ✓ • Asymptotes ← $y = f(x)$
- ✓ • Intercepts both x & y
- ✓ • Critical Points $\rightarrow f'(x) = 0$ or $f'(x) \Rightarrow DNE$
- ✓ • Intervals of INC/DEC ← MAX, MIN NEITHER
- Classify Critical Points
- Inflection Points $f''(x) = 0$ $\begin{matrix} \uparrow \vdots \\ \downarrow \vdots \end{matrix}$ or $\begin{matrix} \uparrow \vdots \\ \uparrow \vdots \end{matrix}$
- Intervals of Concavity

NOTE:

- Use only the essential steps when attempting to sketch a curve.
- Use your previous knowledge of functions whenever possible.

Ex1: Graph $f(x) = \frac{x-4}{x^2-x-2}$ using the curve sketching algorithm.

p.209
(Example 2)

Domain: $f(x) = \frac{x-4}{(x+1)(x-2)}$

$$\{x \in \mathbb{R}, x \neq -1 \text{ or } 2\}$$

VERTICAL ASYMPTOTES

Asympt: @ $x = -1$ & $x = 2$

INTERCEPTS: $f(x) = 0$ (x-ints)

$$0 = x - 4$$

$$x = 4$$

$$(4, 0)$$

y-int ($x=0$)

$$f(0) = 2$$

CRITICAL POINTS: $f'(x) = 0$ & $f'(x)$ DNE

$$f'(x) = \frac{x^2 - x - 2 - (2x-1)(x-4)}{(x^2 - x - 2)^2}$$

$$= \frac{-x^2 + 8x - 6}{(x^2 - x - 2)^2}$$

$$0 = -x^2 + 8x - 6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{64 - 24}}{-2}$$

$$= \frac{-8 \pm \sqrt{40}}{-2}$$

$$= \frac{-8 \pm \sqrt{4 \cdot 10}}{-2}$$

$$= 4 \pm \sqrt{10}$$

$$x = 4 + \sqrt{10} \quad \text{or} \quad x = 4 - \sqrt{10}$$

$$\approx 7.16$$

$$\approx 0.84$$

$$f(7.16) \approx 0.1$$

$$f(0.84) \approx 1.5$$

$$(7.16, 0.1)$$

$$(0.84, 1.5)$$

Asymptotes cont...

@ $x = -1$

$$\lim_{x \rightarrow -1^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -1^+} f(x) = \infty$$

(-1.0001)

@ $x = 2$

$$\lim_{x \rightarrow 2^-} f(x) = \infty$$

$$\lim_{x \rightarrow 2^+} f(x) = -\infty$$

Horizontal Asyms:

$$\lim_{x \rightarrow -\infty} f(x) = 0, y < 0 \quad \lim_{x \rightarrow +\infty} f(x) = 0, y > 0$$

Intervals	$x < -1$	$-1 < x < 0.8$	$0.8 < x < 2$	$2 < x < 7.2$	$x > 7.2$
$-x^2 + 8x - 6$	-	-	+	+	-
$(x^2 - x - 2)^2$	+	+	+	+	+
$f'(x)$	-	-	+	+	-

$$F'(x) = \frac{-x^2 + 8x - 6}{(x^2 - x - 2)^2}$$

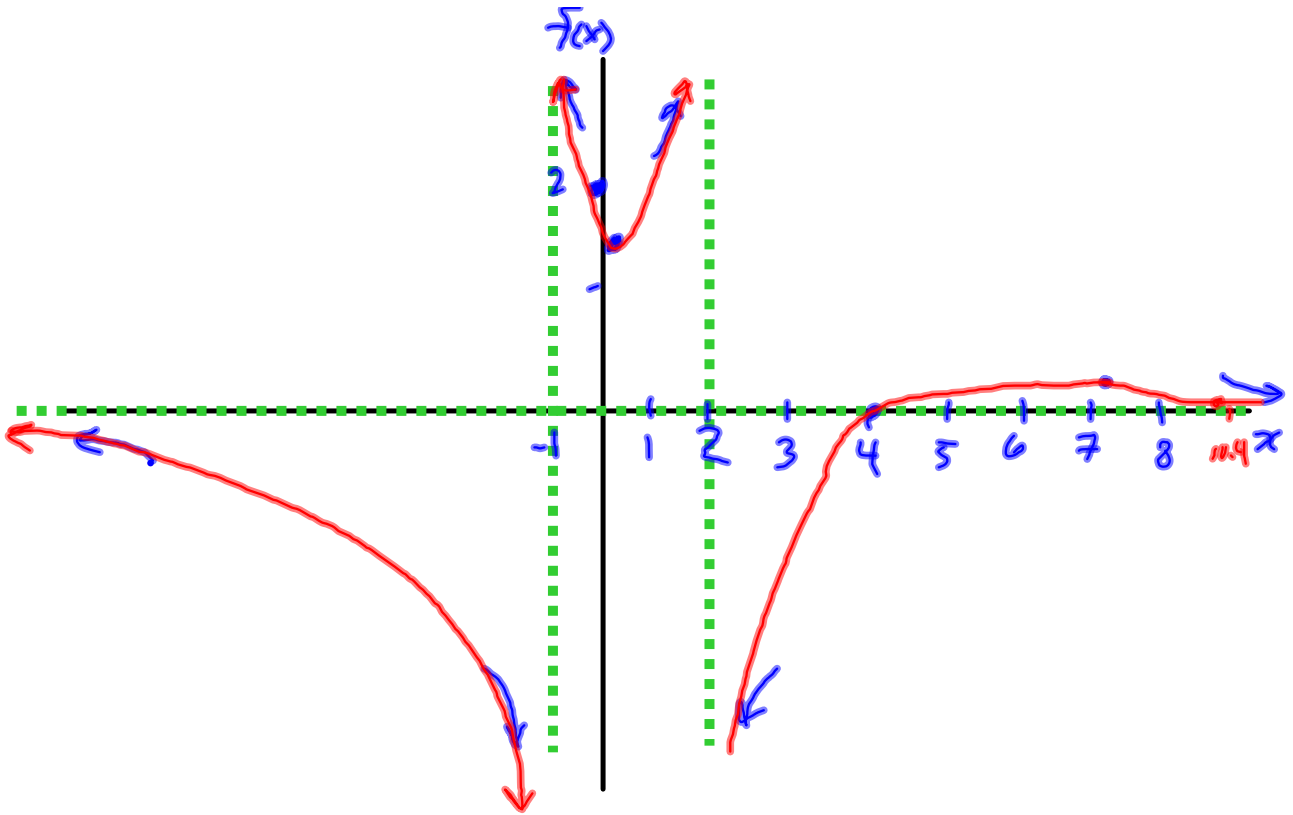
$$F''(x) = \frac{(-2x+8)(x^2-x-2)^2 - 2(x^2-x-2)(2x-1)(-x^2+8x-6)}{(x^2-x-2)^4}$$

$$f''(x) = \frac{2x^3 - 24x^2 + 36x - 28}{(x^2 - x - 2)^3}$$

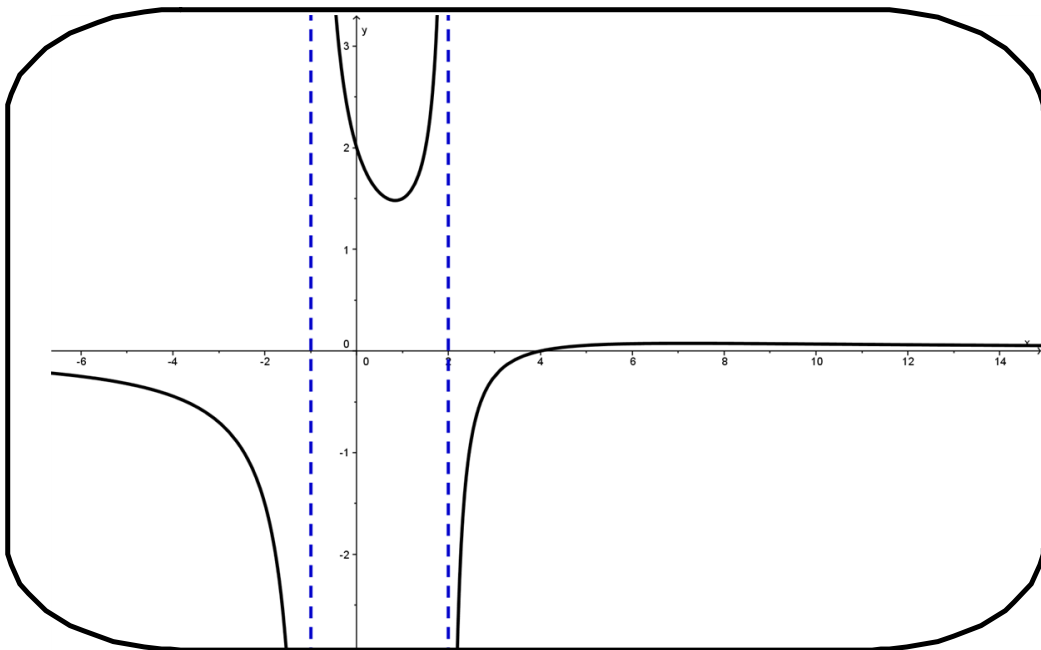
⋮

$$0 = 2x^3 - 24x^2 + 36x - 28$$

⋮



From Geogebra



Assigned Work:

p. 212 # 4abcde

Next Two Periods:

p. 212 # 4fghij