

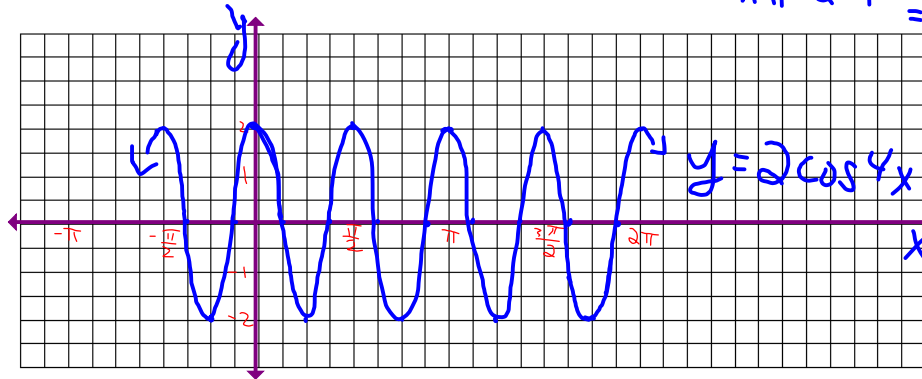
Lesson #5.3: Sinusoidal Functions of the Form

$$f(x) = a \sin[k(x - d)] + c \text{ and } f(x) = a \overset{\text{cos}}{\sin}[k(x - d)] + c$$

Example #1: Determine the amplitude and period of the sinusoidal function.

Then, transform the graph of $y = \cos x$ to sketch a graph of the function: $y = 2 \cos 4x$

Amp = 2 per = $\frac{2\pi}{4} = \frac{\pi}{2}$



Example #2: Consider the function: $y = -2 \sin\left[4\left(x - \frac{\pi}{6}\right)\right] - 3$

a) What is the amplitude?

2

b) What is the period?

$\frac{\pi}{2}$

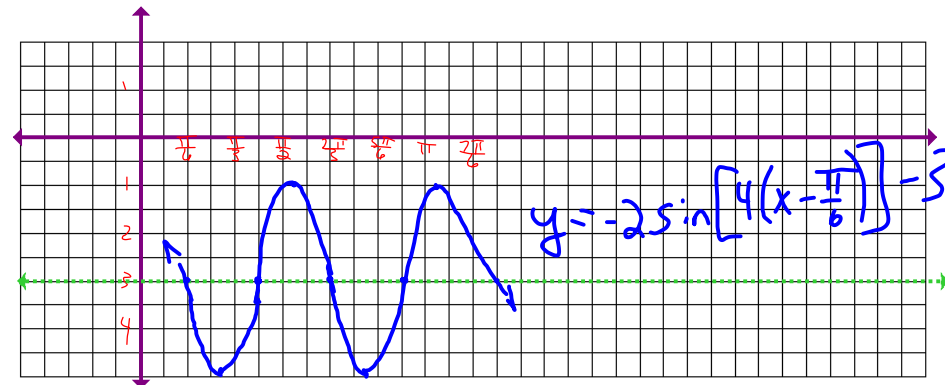
c) Describe the phase shift.

$\frac{\pi}{6}$ (right $\frac{\pi}{6}$)

d) Describe the vertical translation.

-3 (Down 3)

e) Sketch a graph of the function over two cycles.



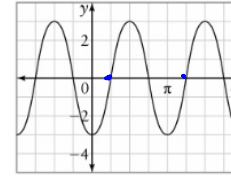
$$\text{period} = \frac{2\pi}{k}$$

$$y = a \sin[k(x-d)] + c$$

Example #3: Model the graph shown using a sine function.

a) From the graph, determine (using the sine function):

- i) the amplitude 3 ii) the period π
 iii) the phase shift $\frac{\pi}{4}$ iv) the vertical translation 0



b) Write an equation for the function:

$$a=3, k=2 \quad d=\frac{\pi}{4} \quad c=0 \quad \text{so } y=3\sin\left[2\left(x-\frac{\pi}{4}\right)\right]$$

c) Graph the function you found in part b) and compare it to the given graph. Verify that the two graphs match.

d) Determine a model for the above graph using a cosine function.

$$y = -3 \cos(2x)$$

e) Verify that the graph of your equation in part d) matches the given graph.

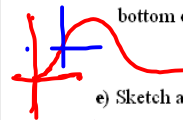
Example #4: A Ferris wheel at an amusement park completes one revolution every 40 s. The wheel has a diameter of 16 m and its centre is 12 m above the ground.

a) If you were to model the height above the ground of a rider with respect to time using a sine function, determine the amplitude and period of the function.
 b) Determine the vertical translation for the function in part a). 12m

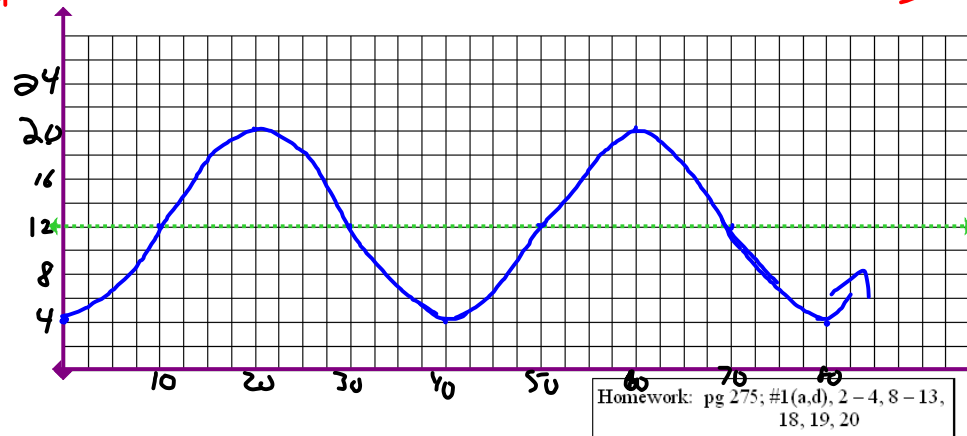
$$\text{amp} = 8 \quad \text{period} = 40\text{s}$$

c) Use graphing technology to determine the phase shift necessary if the rider must enter the ride at the bottom of the wheel.
 d) Model the height, h , in metres, above the ground of a rider using a sine function in the form $h = a \sin[k(t-d)] + c$, where t represents the time, in seconds.

$$h = 8 \sin\left[\frac{\pi}{20}(t-10)\right] + 12$$



e) Sketch a graph of the model over two cycles.



$$\begin{aligned} \text{Per} &= \frac{2\pi}{k} \\ 40 &= \frac{2\pi}{k} \\ k &= \frac{2\pi}{40} \\ &= \frac{\pi}{20} \end{aligned}$$