\[ \mathbf{r} = (-3,5,6) + s(-1,1,2) + t(2,1,-3) \]

\[ \mathbf{BA} = (1,1,-4) \]

\[ \mathbf{r}_1 = (2,2,1) \]

\[ \mathbf{r}_2 = (1,1,5) \]
A linear system of two (or more) equations is said to be CONSISTENT if it has at least one solution, otherwise it is INCONSISTENT.

A linear system may have:

a) No solutions:
   - the lines do not intersect
   - the lines are parallel & distinct (in same plane)
   - the lines may be skew: they are not parallel, but they do not intersect because they lie in different planes

b) 1 Unique solution:
   - the lines cross at one point
   - the angle between two lines is calculated using the dot product between two direction vectors

c) Infinite solutions:
   - the lines are coincident
Ex1: Solve the following system and describe the geometric relationship between the lines.

\[ \begin{align*}
  &a) \quad x = 6 - 18s, \quad y = 12 + 3s \\
  &x = 8 - 6t, \quad y = 4 + 9t
\end{align*} \]

First: Analyze the direction vectors of both lines.

Next: Solve the system by equating \( x \) and \( y \).

### 1. \( 6 - 18s = 8 - 6t \)

\[
\begin{align*}
  6t - 18s - 2 & = 0 \\
  3t - 9s - 1 & = 0 \quad \rightarrow \quad 3t - 9s - 1 = 0
\end{align*}
\]

### 2. \( 12 + 3s = 4 + 9t \)

\[
\begin{align*}
  9s - 3s - 8 & = 0 \quad \rightarrow \quad 9t - 3s - 8 = 0 \\
  20 - 2 & \quad -24s + 5 = 0 \\
  s & = \frac{5}{24}
\end{align*}
\]

Substitute into \( 1 \) to get \( t \)

\[
\begin{align*}
  3t - 9(\frac{5}{24}) - 1 & = 0 \\
  3t & = 1 + \frac{45}{24} \\
  t & = \frac{29}{24}
\end{align*}
\]

\[
\begin{align*}
  x & = 8 - 6t \\
  & = 8 - 6(\frac{29}{24}) \\
  & = 8 - \frac{29}{4} \\
  & = \frac{9}{4}
\end{align*}
\]

\[
\begin{align*}
  y & = 4 + 9t \\
  & = 4 + 9(\frac{29}{24}) \\
  & = 4 + \frac{69}{8} \\
  & = \frac{101}{8}
\end{align*}
\]

:. The point of int

: \( (\frac{9}{4}, \frac{101}{8}) \).

The lines cross at 1

point (1 unique solution)

The system is consistent.
b) \(2x + 5y + 15 = 0\)
\(3x - 4y + 11 = 0\)

\[ \mathbb{R}^2 \]

Gr. 10
\[ y = mx + b \]

Gr. 12
\[ Ax + By + C = 0 \]

\[ \vec{n} = (A, B) \]

\[ C \]

if \( C \)'s are the same, \( \vec{n} \) the same or scalar multiply

\[ 2x + 5y + 10 = 0 \]

\[ 6x + 15y + 30 = 0 \]

COINCIDENT LINES.
c) \[ \frac{x + 4}{3} = \frac{y - 12}{4} = \frac{z - 3}{6} \]

\[ \frac{x}{1/2} = \frac{y - 10}{2/3} = z + 5 \]

\[ \Rightarrow \vec{r}_1 = (-4, 12, 3) + t(3, 4, 6), t \in \mathbb{R} \]

\[ \Rightarrow \vec{r}_2 = (0, 10, -5) + s\left(\frac{1}{2}, \frac{2}{3}, 1\right), s \in \mathbb{R} \]

We can see that these lines are either

- parallel
- coincident

or

\[ \vec{M}_1 = 6\vec{M}_2 \]

So use point (0, 10, -5) in \( \vec{r}_1 \) to see if consistent

\[ \begin{align*}
\text{x-comp} & : 0 = -4 + 3t \\
\text{y-comp} & : 10 = 12 + 4t \\
\text{z-comp} & : \text{skipped} \Rightarrow \text{no need to check}
\end{align*} \]

\[ t = \frac{4}{3} \quad t = -\frac{2}{3} = -\frac{1}{2} \]

\[ \Rightarrow \text{not consistent} \]

\[ \Rightarrow \text{infinite solution} \]

\[ \Rightarrow \text{no solution (parallel)} \]

See. Ex 6 p. 494 For Full solution
d) \[ \vec{r}_1 = (-2,0,-3) + t(5,1,3) \]
\[ \vec{r}_2 = (5,8,-6) + s(-1,2,-3) \]

3. Recognize either
   
   NO SOLUTION (SKEW)
   
   OR
   
   1 SOLUTION

\[ (-2,0,-3) + t(5,1,3) = (5,8,-6) + s(-1,2,-3) \]

\[ \begin{align*}
  x\text{-comp} & : -2 + 5t = 5 - s \\
  y\text{-comp} & : t = 8 + 2s \\
  z\text{-comp} & : -3 + 3t = -6 - 3s \\
\end{align*} \]

Sub in:

\[ -2 + 5(8+2s) = 5 - s \]

\[ -2 + 40 + 10s = 5 - s \]

\[ 11s = -33 \]

\[ s = -3 \]

\[ t = 2 \]

Check \( s = -3 \) & \( t = 2 \) in \( z\text{-components} \):

\[ \begin{align*}
  \text{LS} & = -3 + 3(2) \\
  \text{RS} & = -6 - 3(-3) \\
\end{align*} \]

\[ = 3 \]

\[ = 3 \]

\[ \therefore \text{LS} = \text{RS} \]

or use \( s = -3 \) in \( \vec{r}_2 \)

Using \( t = 2 \):

\[ \vec{r}_1 = (-2,0,-3) + 2(5,1,3) \]

\[ = (8,2,3) \]

\[ \therefore \text{The system is consistent and has one solution, } (8,2,3). \]
Assigned Work:

p.497-498 #8, 9, 11, 12