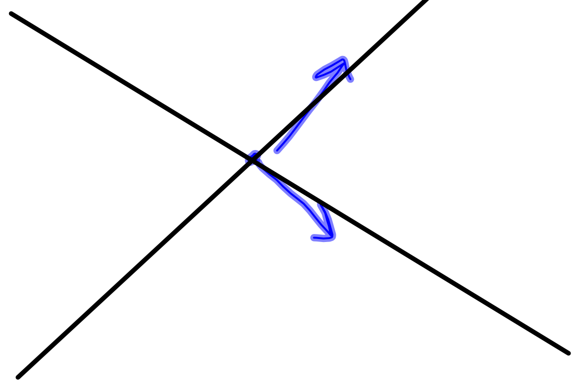
p. 460

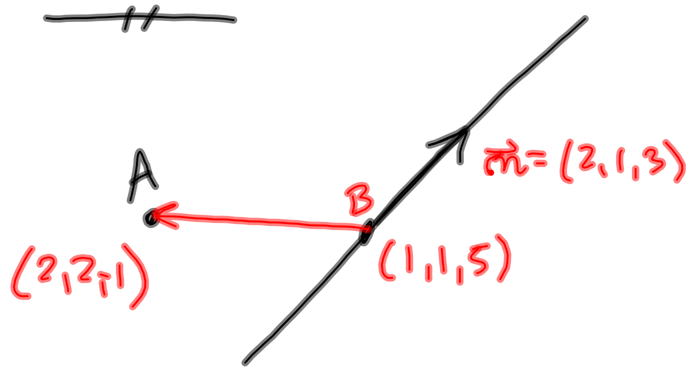
#8

$\pi: \vec{r} = (-3,5,6) + s(-1,1,2) + v(2,1,-3)$

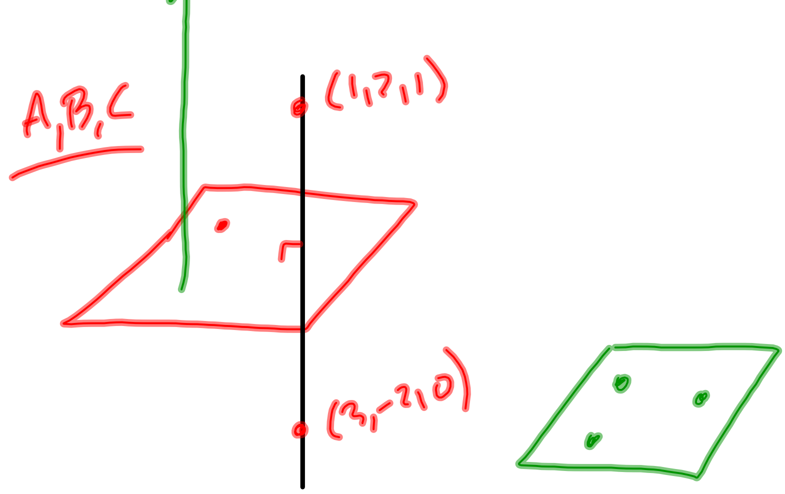


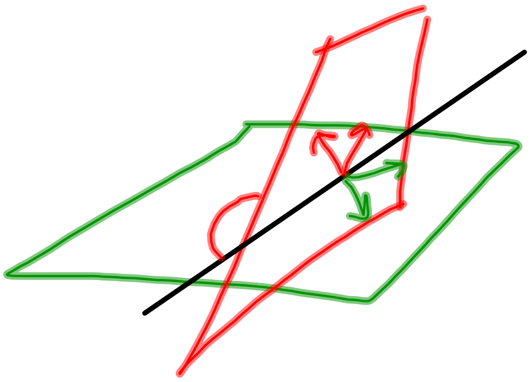
p. 469

#10



$\vec{BA} = (1,1,-6)$





L6(9.1) - The Intersection of Lines in R^3, R^2

A linear system of two (or more) equations is said to be CONSISTENT if it has at least one solution, otherwise it is INCONSISTENT.

A linear system may have:

a) No solutions:
- the lines do not intersect
- the lines are parallel & distinct (in same plane)
- the lines may be **skew**: they are not parallel, but they do not intersect because they lie in different planes

b) 1 Unique solution:
- the lines cross at one point
- the angle between two lines is calculated using the dot product between two direction vectors

c) Infinite solutions: - the lines are **coincident**

Ex1: Solve the following system and describe the geometric relationship between the lines.

a) $x = 6 - 18s, \quad y = 12 + 3s$
$x = 8 - 6t, \quad y = 4 + 9t$

R^2 or R^3 ?

Find point of intersection (x, y)

① $6 - 18s = 8 - 6t$

$6t - 18s - 2 = 0$

$3t - 9s - 1 = 0 \longrightarrow$ ① $3t - 9s - 1 = 0$

② $12 + 3s = 4 + 9t$

$9t - 3s - 8 = 0 \longrightarrow$ ② $9t - 3s - 8 = 0$

3①−②: $-24s + 5 = 0$

$s = \frac{5}{24}$

Sub into ① to get t

$3t - 9\left(\frac{5}{24}\right) - 1 = 0$

$3t - \frac{45}{24} - 1 = 0$

$3t = 1 + \frac{45}{24}$

$t = \frac{23}{24}$

$x = 8 - 6t$
$= 8 - 6\left(\frac{23}{24}\right)$
$= 8 - \frac{23}{4}$
$= \frac{9}{4}$

$y = 4 + 9t$
$= 4 + \cancel{9}^{3}\left(\frac{23}{\cancel{24}_{8}}\right)$
$= 4 + \frac{69}{8}$
$= \frac{101}{8}$

∴ The point of int is $\left(\frac{9}{4}, \frac{101}{8}\right)$.

The lines cross at 1 point (1 unique solution)

The system is consistent.

R^2

GR.10

$y = mx + b$

GR.12

$Ax + By + C = 0$

b) $2x + 5y + 15 = 0$
$3x - 4y + 11 = 0$

$\vec{n} = (A, B)$ C

if C's are the same & $\vec{n}$ the same or scalar multiply

$$\left.\begin{aligned} 2x + 5y + 10 &= 0 \\ 6x + 15y + 30 &= 0 \end{aligned}\right\}$$ COINCIDENT LINES.

c) $\dfrac{x+4}{3} = \dfrac{y-12}{4} = \dfrac{z-3}{6}$ $\Rightarrow \vec{r}_1 = (-4, 12, 3) + t(3, 4, 6), t \in \mathbb{R}$

$\dfrac{x}{1/2} = \dfrac{y-10}{2/3} = z + 5$ $\Rightarrow \vec{r}_2 = (0, 10, -5) + s\left(\frac{1}{2}, \frac{2}{3}, 1\right), s \in \mathbb{R}$

We can see that these lines are either

- parallel
  OR
- coincident

$\left.\right\}$ $\because \vec{m}_1 = 6\vec{m}_2$

$(3,4,6) = 6\left(\frac{1}{2}, \frac{2}{3}, 1\right)$

So use point $(0,10,-5)$ in $\vec{r}_1$ to see if consistent

x-comp	y-comp	z-comp
$0 = -4 + 3t$	$10 = 12 + 4t$	skipped $\rightarrow$ no need to check
$t = \frac{4}{3}$	$t = -\frac{2}{4} = -\frac{1}{2}$	$\therefore$ NOT CONSISTENT

$\rightarrow$ ~~infinite solution~~

no solution (parallel)

See. Ex 6 p. 494 for Full solution

d) $\vec{r_1} = (-2,0,-3) + t(5,1,3)$
$\vec{r_2} = (5,8,-6) + s(-1,2,-3)$

Recognize either
NO SOLUTION (SKEW)
OR
1 SOLUTION

$(-2,0,-3)+t(5,1,3) = (5,8,-6) + s(-1,2,-3)$

x-comp
$-2 + 5t = 5 - s$

y-comp
$t = 8 + 2s$

z-comp
$-3+3t = -6 - 3s$

sub in

$-2+5(8+2s) = 5-s$

$-2 + 40 + 10s = 5 - s$

$11s = -33$

$s = -3$

$t = 2$

Check $s=-3$ & $t=2$ in z-components:

LS	RS
$-3+3(2)$	$-6-3(-3)$
$= 3$	$= 3$

$\therefore$ LS = RS

OR use $s=-3$ in $\vec{r_2}$

using $t=2$
$\vec{r_1} = (-2,0,-3) + 2(5,1,3)$
$= (8,2,3)$

$\therefore$ The system is consistent and has one solution, $(8,2,3)$.

Assigned Work:

p.497-498 #8, 9, 11, 12