L6 (3.4) Optimization Problems in Economics

NOTE: 1- Profit = Revenue - Cost or P = R - C

2- Revenue = (price per unit)(number of units)

3- The cost function will sometimes be given in the problems we do in this course.

Ex1: A professional basketball team plays in an arena that holds 20 000 spectators. Average attendance at each game has been 14 000. The average ticket price is $75. Market research shows that for each $5 reduction in ticket price, attendance increases by 800 people.
Find the price that will maximize revenue.

Revenue = (price per unit)(# of units sold)

Let x rep the number of price changes.

$R(x) = (75 - 5x)(14000 + 800x) \quad \{-17.5 < x < 7.$

$R'(x) = -5(14000 + 800x) + 800(75 - 5x)$

Set $R'(x) = 0$ to find crit values.

$5(14000 + 800x) = 800(75 - 5x)$

$17.5 + x = 15 - x$

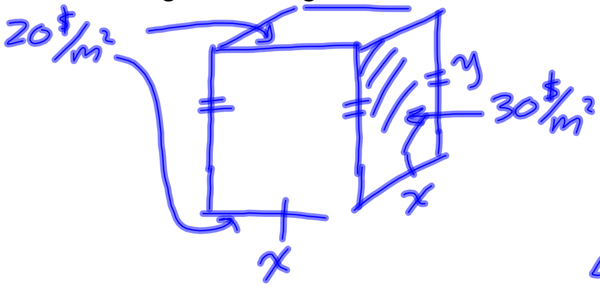
$2x = -2.5$

$x = -1.25$ ← possible increase in pr

$\text{Price} = 75 - 5(-1.25) \qquad R(-1.25) = 10562$

$= 81.25$

Ex2: Chris is constructing a closed rectangular box with a square base from two different types of wood. The wood for the top and bottom costs \$20/m². The wood for the sides costs \$30/m². Find the dimensions that minimize the cost of building the box given that its volume is 4000 m³.



$V = 4000 m^3$

$V = l \cdot w \cdot h$

$4000 = x \cdot x \cdot y$

$y = \frac{4000}{x^2}$

$$C(x) = 2x^2(20) + 4x\left(\frac{4000}{x^2}\right)(30)$$

$$= 40x^2 + \frac{480000}{x}$$

$$= 40x^2 + 480000x^{-1}$$

$$C'(x) = 80x - 480000x^{-2}$$

Set $C'(x) = 0$ to find crit. values.

$$\frac{480000}{x^2} = 80x$$

$$x^3 = \frac{480000}{80}$$

$$= 6000$$

$$x \doteq 18.17$$

$$y = \frac{4000}{x^2}$$

$$\doteq \frac{4000}{(18.17)^2}$$

$$\doteq 12.11$$

∴ The dimensions that will minimize the cost are $18.17 \times 18.17 \times 12.11$ m.

Ex3: A local furniture manufacturer sells cedar patio sets. The company can sell x units each month at a price p = 1000 - x in dollars, where the cost, C, of production is $C(x) = 3000 + 19x^2$. Determine the price that will maximize profits.

✓

$$P_{(x)} = R_{(x)} - C_{(x)}$$

$$= (\text{price per unit})(\#\text{ of units}) - C_{(x)}$$

$$= (1000 - x)(x) - [3000 + 19x^2]$$

$$= 1000x - x^2 - 3000 - 19x^2$$

$$= -20x^2 + 1000x - 3000$$

$$P'_{(x)} = -40x + 1000$$

Set $P'_{(x)} = 0$ to find crit. values.

$$40x = 1000$$

$$x = 25$$

To determine the price:

$$\text{Price} = 1000 - x$$

$$= 1000 - 25$$

$$= 975$$

∴ The price that would maximize profits is \$975.

Assigned Work:

p. 152-153 # 2, 3, 4, 5, 6, 7, 10, 12, 13