# Lesson #6.4: Power Law of Logarithms

Example #1: Evaluate each of the following:

a) $\log_2 8 = 3$ b) $\log_3 81 = \log_3 3^4 = 4$ c) $\log_5 125 = \log_5 5^3 = 3$ d) $\log 100 = \log 10^2 = 2$

| | | | |
|---|---|---|---|
| Let $y = \log_2 8$<br>$2^y = 8$<br>$2^y = 2^3$<br>So $y = 3$ | Let $x = \log_3 81$<br>then $3^x = 81$<br>$3^x = 3^4$<br>$x = 4$ | Let $a = \log_5 125$<br>So $5^a = 125$<br>$5^a = 5^3$<br>$\therefore a = 3$ | Verify on TI |

**In General:** $\log_b b^n = n$.

Example #2: Evaluate each of the following:

| | |
|---|---|
| $\log 2^1$ $\log 2 = 0.30103$ | |
| $\log 2^2$ $\log 4 = 0.60206$ | |
| $\log 2^3$ $\log 8 = 0.90309$ | |
| $\log 2^4$ $\log 16 = 1.20412$ | |
| $\log 2^5$ $\log 32 = 1.50515$ | |
| $\log 2^n = n(\log 2)$ | |

| | |
|---|---|
| $\log 3 = 0.47712$ | |
| $\log 9 = 0.95424$ | |
| $\log 27 = 1.43136$ | |
| $\log 81 =$ | : |
| $\log 243 =$ | : |
| $\log 3^n = n \log 3$ | |

To prove this rule, let $w = \log_b x$, ←

so $x = b^w$

then $x^n = (b^w)^n$

$= b^{nw}$

$\log_b x^n = nw$

so $\log_b x^n = n \log_b x$

**Power Law of Logarithms:**

$\log_b x^n = n \log_b x$, $b > 0, b \neq 1, n \in \Re$

Example #3: Use the power law to evaluate without using a calculator.

a) $\log_2 4^{-5}$

$= -5 \log_2 4$

$= -5(\log_2 2^2)$

$= -5(2)$

$= -10$

b) $\log_3 27^7$

$= \log_3 (3^3)^7$

$= \log_3 3^{21}$

$= 21$

c) $\log_5 \sqrt[3]{25}$

$= \log_5 (25)^{\frac{1}{3}}$

$= \frac{1}{3} \log_5 5^2$

$= \frac{2}{3}$

d) $\log_8 \sqrt{2}$

$= \log_8 2^{\frac{1}{2}}$

$= \frac{1}{2} \log_8 2$

$= \frac{1}{2} \log_{8} 8^{\frac{1}{3}}$

$= \frac{1}{2}\left(\frac{1}{3}\right)$

$= \frac{1}{6}$

$2 = 8^{\frac{1}{3}}$

Example #4:
Solve for $n$, correct to three decimal places.

$2000 = 500(1.045)^n$

$4 = 1.045^n$

$\log 4 = \log(1.045)^n$

$\log 4 = n \log(1.045)$

$\frac{\log(4)}{\log 1.045} = n$

$31.495 \doteq n$

This same technique can be generalized to determine the value of logarithms of any base, b>0, b ≠ 1

**Let** $x = \log_b m$

**then** $b^x = m$

$\log b^x = \log m$

$x \log b = \log m$

**so** $x = \frac{\log m}{\log b}$

Write in exponential form and take the common logs of both sides

So $\log_b m = \frac{\log m}{\log b}$

To calculate a logarithm with any base, express in terms of common logarithms using the **change of base formula**:

$$\log_b m = \frac{\log m}{\log b}$$

Example #5: Evaluate:

$\log_9 12 = \frac{\log 12}{\log 9}$

$\doteq 1.1$

Example #6: Write as a single logarithm, then evaluate:

$\frac{\log 16}{\log 4} = \log_4 16$

$= \log_4 4^2$

$= 2$

Example #7: Solve each of the following, correct to 2 decimal places:

a) $\log 4^x = 7$

$x \log 4 = 7$

$x = \frac{7}{\log 4}$

$\doteq 11.63$

b) $12 = \log_3 4^m$

$12 = m \log_3 4$

$\frac{12}{\log_3 4} = m$ $\Rightarrow 12 = m \frac{\log 4}{\log 3}$

$\frac{12}{\frac{\log 4}{\log 3}} = m$ $\frac{12 \log 3}{\log 4} = m$

$\frac{12 \log 3}{\log 4} = m$

$9.51 \doteq m$

Homework: pg 347; #1 – 7, 9, 10