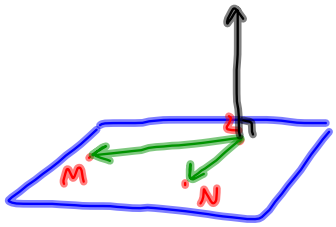
Assessment Quiz 5 Unit 7

Find the Cartesian Equation of the plane defined by the points L(1,3,6) M(-3,5,8) and N(-4,6,-6).



$\vec{LM} = (-4, 2, 2)$ $\vec{LN} = (-5, 3, -12)$

$\vec{n} = \vec{LM} \times \vec{LN}$

$$= \begin{vmatrix} i & j & k \\ -4 & 2 & 2 \\ -5 & 3 & -12 \end{vmatrix}$$

$= (-30, -58, -2)$

Sub $\vec{n}$ & (1,3,6) into $Ax + By + Cz + D = 0$ to find D:

$-30(1) - 58(3) - 2(6) + D = 0$

$D = 216$

$\therefore$ The Cartesian Eqn of π is $-30x - 58y - 2z + 216 = 0$

- OR -

$15x + 29y + z - 108 = 0$

L7(9.1) - Intersections of a Line and a Plane

There are three possibilities:

1 - No solution	The line and the plane are paralle**ho point of intersection.**
2 - One solution	The line cuts the plane,**one point of intersection.**
3 - Infinite solutions	The line is in the plane,**infinite points of intersection described by the equation of the line.**

Ex1: Find the intersection of the line and the plane

L: $\vec{r} = (4,6,-2) + t(-1,2,1)$ $\rightarrow \vec{m} = (-1,2,1)$

π: $2x - y + 6z + 10 = 0$ $\rightarrow \vec{n} = (2,-1,6)$

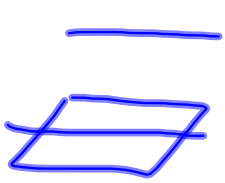
Check possibilities using dot product:

$\vec{n}_\pi \cdot \vec{m}_L = (2,-1,6) \cdot (-1,2,1)$

$= -2 - 2 + 6$

$= 2$

$\Rightarrow \because$ not equal to zero



Step 1: get parametric form of L

$x = 4 - t$

$y = 6 + 2t$

$z = -2 + t$

Step 2: Sub x, y, z into π to determine t.

$2x - y + 6z + 10 = 0$

$2(4-t) - (6+2t) + 6(-2+t) + 10 = 0$

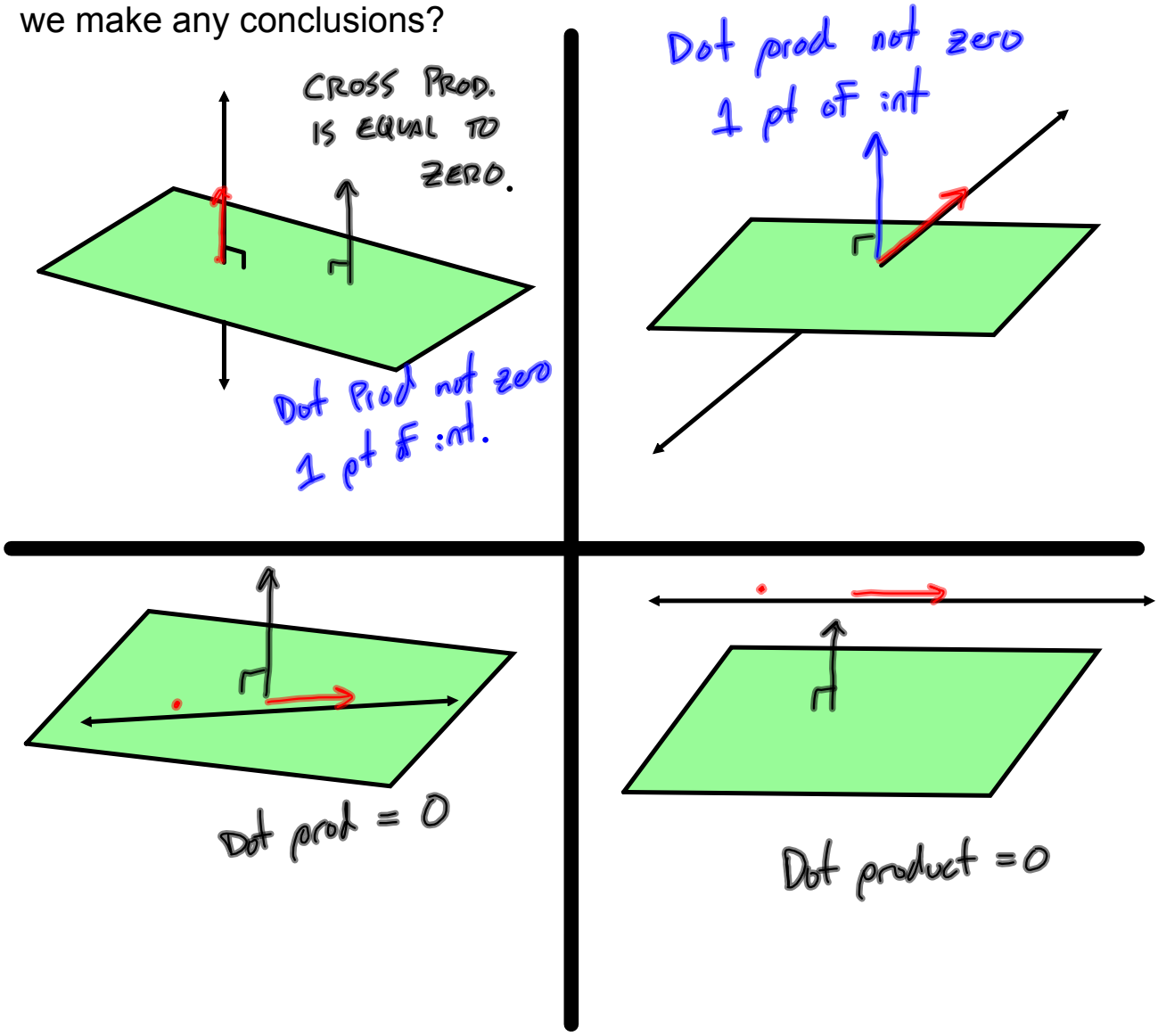
$8 - 2t - 6 - 2t - 12 + 6t + 10 = 0$

$2t = 0$

$t = 0$

$\therefore$ the point of intersection is $(4,6,-2)$.

Draw the direction vector of the line and the normal of the plane. Can we make any conclusions?



Ex2: Find the intersection of the line and the plane

L: $x = 5 + t$ and π: $2x + 3y - 4z - 7 = 0$
$y = 4 + 2t$
$z = 7 + 2t$

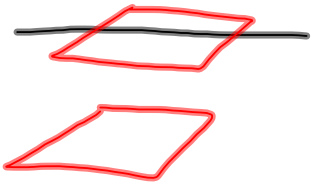
$2(5+t) + 3(4+2t) - 4(7+2t) - 7 = 0$

$10 + 2t + 12 + 6t - 28 - 8t - 7 = 0$

$0t = 13$

∴ There are no solutions for t

∴ No points of intersection ∴ the line is parallel to the plane



what if

$0t = 0$

$t \in \mathbb{R}$ ∴

infinite solutions defined by the line L.

Assigned Work:

p.496-498 # 3, 4, 5, 6, 7, 13

Review

Read Ch 8 Summary p.479

Review p.480-483 (34 questions)

Chapter 9.1 (Intersections of L & L/ L & Plane)