L9 - Matrices

Example 1: Solve the system of equations by using an augmented matrix and Gauss-Jordan Elimination.

$$\begin{aligned} x - 3y - 2z + 9 &= 0 \\ 2x - 5y + z - 3 &= 0 \\ -3x + 6y + 2z - 8 &= 0 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \# \\ 0 & 1 & 0 & \# \\ 0 & 0 & 1 & \# \end{array}\right]$$

When leading 1's are produced along the diagonal, along with zeros below the 1's, we say the matrix is in *row echelon form.*

$$\left[\begin{array}{ccc|c} 1 & \# & \# & \# \\ 0 & 1 & \# & \# \\ 0 & 0 & 1 & \# \end{array}\right]$$

When leading 1's are produced along the diagonal, and all values above & below the 1's are zeros, we say the matrix is in reduced *row echelon form.*

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \# \\ 0 & 1 & 0 & \# \\ 0 & 0 & 1 & \# \end{array}\right]$$

In general, we can get a matrix into row echelon form in about four steps:

1- If possible, rearrange rows to get a 1 in the upper left corner (*First equation has a coefficient of x equal to 1*).

2- Use Gauss-Jordan Elimination to get zero's under the top left number.

$$\begin{array}{c} ① \\ ② \\ ③ \end{array} \left[\begin{array}{ccc|c} 1 & -3 & -2 & -9 \\ 2 & -5 & 1 & 3 \\ -3 & 6 & 2 & 8 \end{array}\right] \quad \begin{array}{c} ① \\ ①\times2-② \\ ①\times3+③ \end{array} \left[\begin{array}{ccc|c} 1 & -3 & -2 & -9 \\ 0 & -1 & -5 & -21 \\ 0 & -3 & -4 & -19 \end{array}\right]$$

3- Use Gauss-Jordan Elimination to get zero under the middle number.

$$\begin{array}{c} \\ ②\times-1 \\ ②\times3-③ \end{array} \left[\begin{array}{ccc|c} 1 & -3 & -2 & -9 \\ 0 & 1 & 5 & 21 \\ 0 & 0 & -11 & -44 \end{array}\right]$$

4- Divide the rows to produce leading 1's.

Note:

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & -9 \\ 0 & 1 & 5 & 21 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Once the matrix is in $\boxed{\text{row echelon form}}$ the solution to the system can be found. We can easily find the value of z and back solve for y and x.

It is possible for a matrix to give

1- one solution (system would be consistent)

2- infinite solutions (system would be consistent)

3- no solution (system would be inconsistent)

①

$$3x + 2y + z = 0$$
$$6x + 4y + 2z = 0$$
$$3x + 9y + 12z = 0$$

②

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 6 & 4 & 2 & 0 \\ 3 & 9 & 12 & 4 \end{array}\right]$$

$$①\times3-② \left[\begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Example 2: Express the following matrix in row echelon form and find the values of x, y, & z that would satisfy the system.

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 1 & -3 & 2 & -10 \\ 3 & 1 & -1 & -4 \end{array}\right]$$

Solution:

x = -46/25, y = 128/25, z = 18/5

HW

p. 309 #7 , 8a